Review of analysis methods for inelastic design of steel semi-continuous frames

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Abstract

Three groups of methods for the inelastic analysis of steel plane frames with semi-rigid joints are considered in this paper. The first one consists of simplified second-order (or $P\!-\!\Delta$) plastic-hinge methods for the determination of member internal forces in semi-continuous frames subjected to moderate sway deformations. For these methods, a set of notional loads to simulate second-order effects in the first-order plastic-hinge analysis is used. The second group of methods presented in the paper refers to the general second-order methods of analysis. These methods are based on the concept of a refined plastic hinge in the form of a two-surface degradation model or a spring-in-series model. Refined plastic-hinge methods allow the simulation of the combined effect of gradual joint-stiffness degradation, and distributed plasticity along the member length as well as across the member sections. Finally, the third group consists of more general advanced analysis methods. They are based on second-order refined plastic-hinge methods of analysis in which the effects of residual stresses and geometric imperfections of individual members are accounted for in the global analysis. This type of analysis has recently become more important, since modern design codes (e.g. Eurocode 3: part 1.1) now require the structural engineer to address more rigorously instability problems of real structural systems. An illustrative example is presented. Conclusions are drawn regarding methods of analysis currently used in the design of sway frames, and their future development.

Keywords: Semi-rigid joint; Steel frame; $P\!-\!\Delta$ analysis; Second-order analysis; Advanced analysis; Limit states design; Uniform reliability index

1. Introduction

Present design codes are based on the limit-state design philosophy. When considering structural behaviour at the ultimate limit state, collapse behaviour is of primary importance. The real behaviour of framed structures is, in general, nonlinear and of a rather complex nature. Since the introduction of limit-state design philosophy in structural codes, day-to-day design methodology has not changed much from that of the old permissible-stress design concept. According to earlier philosophy, designers still use linear-elastic structural analysis in which buckling resistance is based on the effective length rather than on the ultimate strength. This type of approach, referred to as the first-plastic-hinge concept, does not reflect the true collapse behaviour in design.

The first algorithmic approach to the first-order plastic-hinge collapse analysis of steel frames was reported by Horne [1], while Backer [2] created the basis for plastic design theory. With the progress of computerized calculations, the method of collapse analysis has subsequently been modified by others and recommended, with some limitations, for practical purposes. The limitations set for the first-order analysis come from the fact that the evaluation of plastic redistribution of bending moments and the failure mechanism are controlled not only by the frame topology

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and physical properties of members and joints, but also (in the case of moment redistribution) by the change of frame geometry. Since routine analyses in design offices do not involve a great deal of research study, the developed new plastic design procedures for use in daily engineering practice were based on simplified geometrically nonlinear methods of analysis. Simplified second-order plastic-hinge analyses that have been identified for engineering applications allow for using conventional first-order plastic-hinge methods, with some modifications to account for change in geometry (e.g. P–Δ methods recommended in ECCS Publication [3]).

Through worldwide research on semi-rigid joints and their influence on frame behaviour, engineers have been given an opportunity to adopt technologically preferred flexible joints. The application of flexible joints has resulted in the design and erection of semi-continuous structural frame systems. The practical use of plastic-hinge concepts for semi-continuous frames has been adopted in design specifications such as ECCS Publication [4], and recently developed modern design codes (e.g. Eurocode 3: Part 1.1). For the plastic design of semi-continuous sway frames, inelastic second-order methods (plastic hinge or, preferably, refined plastic hinge) should generally be used in combination with curvilinear joint characteristic to account for the stiffness degradation effects in members and semi-rigid joints. For frames subjected to moderate sway deformations, simplified methods (with indirect allowance for second-order effects and initial sway imperfections) are suggested.

The methods discussed in the paper are restricted to second order methods of fully laterally restrained frames comprising of class 1 sections for which local instability and lateral torsional buckling do not affect their structural performance. Methods of a higher level of sophistication, based on more accurate strain–displacement equations than those used in the second order analysis, and based on detailed stress–strain relationships at the material point, are not within the scope of this paper.

2. Simplified second order methods

First order elastic or rigid plastic methods of analysis may be used for the assessment of structural performance, if any change of structural behaviour caused by second order effects can be neglected. Although second order elastic or plastic analyses can always be used, there are cases where most recent codes (e.g. Eurocode 3: part 1.1) recommend the use of first order analysis on the condition that second order sway effects are accounted for by increasing the horizontal loads. These effects are referred to as the equivalent horizontal loads accommodating P–Δ effects resulting from vertical loads and initial sway imperfections. The limitation usually imposed on the use of first order analysis excludes the existence of slender members for which initial bow imperfections should have been accounted for. The notional loads for the simplified second order analysis may be identified as the sum of horizontal external and equivalent loads increased by the amplification factor η (e.g. see Eurocode 3: part 1.1):

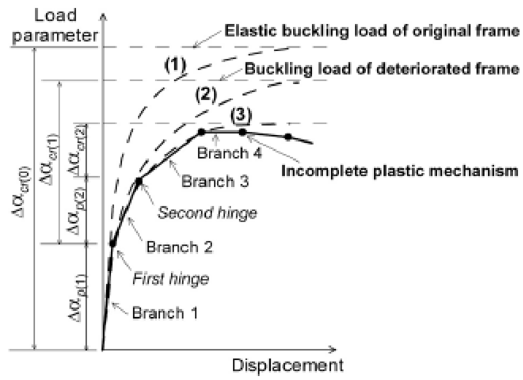

Fig. 1. Second order load deflection characteristic of an inelastic frame.

$$\eta = \frac{1}{1 - \frac{1}{\alpha_{cr}}} \leq 1.5 \tag{1}$$

where α_{cr} is the factor by which the design loading would have to be increased to cause elastic instability in a global sway mode. The amplification factor given by Eq. (1) is applied to single storey frames and to stories of multi-storey frames, if the latter fulfil certain conditions. The said factor is treated as a constant for the purpose of elastic design. In the case of plastic design, a more general approach is needed.

Let us first examine the frame response in the incremental second order plastic hinge analysis in order to explain the frame stability behaviour deteriorated by successive formation of plastic hinges (see Fig. 1).

In stage 1 an elastic second order analysis is first performed and from that the first increment of the load factor $\Delta\alpha_{p(1)}$ is determined. At the load level corresponding to this load factor increment, the first plastic hinge is determined to form in the member end section (or sections in the case of the formation of multiple plastic hinges) or in the member end joint(s). One has to note that in the case of second order elastic analysis of the original frame "F0" the analysis can be continued above the load level corresponding to the load factor $\alpha_{p(1)} = \Delta\alpha_{p(1)}$. This is indicated by the dotted line (1) in Fig. 1. The elastic equilibrium path (1) approaches asymptotically the load level corresponding to the load factor $\alpha_{cr(0)} = \Delta\alpha_{cr(0)}$ that can be calculated from the bifurcation analysis of the original elastic frame "F0".

In stage 2 the analysis is to be performed for the modified frame that is obtained by the introduction of the first mechanical hinge(s) at the point(s) of plastic hinge formation. The modified frame "F1" is called the first deteriorated frame. Formation of the first plastic hinge(s) either in member end joint(s) or/and in member end section(s) produces the plastic rotation in the hinge(s). Provided that the joint(s) or/and the section(s) with the plastic hinge(s) exhibit the rotation capacity sufficient for the plastic redistribution of bending moments, this process may be continued under successive load increments. By performing the elastic second order analysis of the deteriorated frame "F1", the load factor $\Delta\alpha_{p(2)}$ is determined. At the load level corresponding to the load factor $\alpha_{p(2)} = \alpha_{p(1)} + \Delta\alpha_{p(2)}$, the formation of next plastic hinge(s) occurs. The branch 2 of the frame load–deflection characteristic, between the formation of the first and the second plastic hinges, can be related to the second order load–deflection curve of the elastic deteriorated frame "F1". The members of the so-called deteriorated frame are subjected to a constant set of axial forces found at the end of the first load increment $\Delta\alpha_{p(1)}$ and axial forces corresponding to increasing external loads. One has to again note that in the case of elastic second order analysis of deteriorated frame "F1" the analysis can be continued above the load level corresponding to $\alpha_{p(2)} = \alpha_{p(1)} + \Delta\alpha_{p(2)}$. This is indicated by the dotted line (2) in Fig. 1. The elastic equilibrium path of the deteriorated frame "F1" approaches asymptotically the load level corresponding to $\alpha_{cr(1)} = \alpha_{p(1)} + \Delta\alpha_{cr(1)}$. The critical load parameter $\Delta\alpha_{cr(1)}$ of the deteriorated frame "F1" can be obtained from the elastic bifurcation analysis of the deteriorated frame "F1". Such an analysis has to be conducted for the frame "F1" subjected to a set of constant axial forces ("prestress" set of forces) and the set of axial forces obtained from the static analysis of deteriorated frame "F1" ("critical" set of forces). The former is fixed at the end of the first load increment while the latter is dependent on the load parameter α and its critical value is denoted by $\Delta\alpha_{cr(1)}$. The procedure is similar to that of buckling analysis of frames composed of elements with piecewise linear stress–strain characteristic [5].

The incremental analysis described above has to be repeated for the frame progressively deteriorating until the failure mechanism forms at the ultimate state u. The ultimate load corresponds to the load factor $\alpha_u = \alpha_{p(u)}$ at which the last deteriorated frame "Fu" is characterised by a critical load factor $\Delta\alpha_{cr(u)} \leq 0$. This means that the states of internal forces and external loads diverge and the equilibrium under increased deformations can only be maintained under the decreasing loads. The buckling analysis therefore gives $\alpha_{cr(u)} = \alpha_{p(u)} + \Delta\alpha_{cr(u)} \leq \alpha_{p(u)}$. Furthermore, it is indicative of an incomplete plastic mechanism that involves less plastic hinges than that produced by the first order plastic hinge analysis.

An understanding of the frame deterioration process in the second order plastic hinge analysis helps in the development of simplified second order analyses. One such analysis has been proposed by Branicki et al. [6] for simplified tracing of inelastic moment redistribution process in semi-continuous frames subjected to moderate sway deformations. In this simplified approach illustrated in Fig. 2, the first order analysis has to reproduce as closely as possible the results from the second order analysis. The second order equilibrium piecewise curvilinear paths of

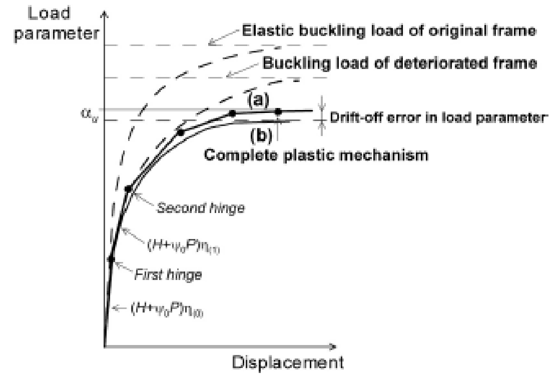

Fig. 2. Simplified second order load deflection characteristic of an inelastic frame.

inelastic sway frames [see path (b) in Fig. 2] are approximated by piecewise linear ones [see path (a) in Fig. 2]. Each linear branch of the frame load–deflection curve identifies the stable equilibrium state between the points of the formation of two successive plastic hinges (or successive sets of multiple plastic hinges). It is obtained from the geometrically linear analysis carried out for a particular deteriorated frame subjected to a set of notional loads applied in the direction of sway degrees of freedom.

In order to account for the second order effects in frames subjected to horizontal sway, the notional loads $H+\psi_0 P$ are to be increased in consecutive stages. The H stands here for external loads, P for the gravity loads and ψ_0 for the initial sway deformations. As the sensitivity of each deteriorated frame to buckling is different, the load amplification factor may not be kept constant. It is therefore suggested that at each load increment i, the notional loads composed at each storey of external loads H and equivalent loads due to initial sway imperfections $\psi_0 P$, are replaced by their amplified values given by:

$$(H + \psi_0 P)\eta_{(i)} \tag{2}$$

where $\eta_{(i)}$ = amplification factor calculated as follows:

$$\eta_{(i)} = \frac{1}{1 - \frac{1}{\Delta\alpha_{cr(i)}}}. \tag{3}$$

Eq. (3) is similar to Eq. (1). Notional loads in the simplified inelastic P–Δ analysis therefore need to be recalculated at each load increment as products of notional loads corresponding to sway degrees of freedom and the amplification factor obtained from stability analysis of the deteriorated frame evaluated for the current stage of analysis. For the evaluation of incremental notional loads, the critical load parameter $\Delta\alpha_{cr(i)}$ of the deteriorated frame is calculated with respect to the set of real loads H. For the first increment $\Delta\alpha_{cr(i)} = \Delta\alpha_{cr(0)} = \alpha_{cr}$ and the critical load factor is calculated for the original frame. The corresponding amplification factor $\eta_{(i)} = \eta_{(0)}$ produces the same set of frame stress resultants as obtained with Eq. (1). It can be immediately noted that the simplified second order analysis proposed by Branicki et al. [6] is a natural extension of the simplified elastic second-order analysis of current design codes (e.g. Eurocode 3: part 1.1). An important feature of the proposed method is that it does not require iterations to be performed within the load increments unlike other simplified iterative second order analyses (e.g. ECCS Publication [3]). The proposed method is based on a simple incremental first order plastic hinge analysis that uses the secant stiffness of semi-rigid joints as specified in current design specifications and amplified notional loads applied in the direction of sway degrees of freedom. Amplified notional loads represent the $P\text{–}\Delta$ effect on bending moments, the frame load deflection characteristic and the frame ultimate state. The joint secant stiffness is taken as a half of the initial one (as recommended by Eurocode 3: part 1.8). It is proposed that this simplified second order analysis has its own range of application up to the load level corresponding to failure load evaluated from the Merchant-Rankine criterion [7].

3. Second order methods

3.1. Conventional plastic hinge model

The second order method of analysis refers to the stability function based analysis, or preferably to the finite element analysis in which the stiffness matrix is the sum of the incremental constitutive stiffness component (in particular the initial stiffness for the elastic range) and the geometric initial stress stiffness component depending linearly on the member axial force. The latter stiffness component (FEM term) allows for the inclusion of geometrically nonlinear $P\text{–}\delta$ and $P\text{–}\Delta$ effects. Since the frame equilibrium path is nonlinear, response analysis has to be performed in a step-by-step fashion, increasing the loads incrementally and monitoring the stiffness softening and bending moment redistribution processes.

Adopting the conventional plastic hinge concept, the joint behaviour can be represented by a frictionless spring of an elastic–ideal-plastic moment–rotation characteristic (see Fig. 3(a)). The following notation is used in Fig. 3(a): $M =$ moment carried out by the joint; $\phi =$ local rotation resulting from the deformability of joint components; $M_{j,R} =$ joint ultimate moment capacity; $S_j =$ joint stiffness. Similarly, the section behaviour in the analysis is described by a rigid–plastic moment–rotation characteristic (see Fig. 3(b)). The following notation holds in Fig. 3(b): $M =$ moment carried out by the section; $\phi =$ rotation of the member ends cut by the section at which the plastic hinge forms; $M_{s,R} =$ section moment resistance, if necessary reduced due to the presence of axial force. It is important to note that the plastic hinge can be notionally treated as a frictionless member section spring with the initial stiffness constant $S_s = \infty$ and the section moment capacity $M_{s,R}$.

When the physical frame is replaced by its discretised finite element model for the purpose of analysis, each finite

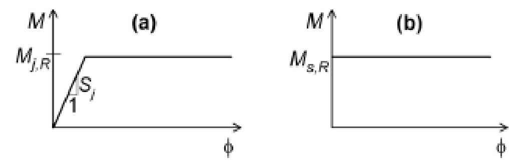

Fig. 3. Plastic hinge moment rotation characteristics; (a) member section, (b) member end joint.

element can be treated in the second order plastic hinge analysis as a superelement having a single end spring with integrated properties evaluated as follows:

$$S_{\text{ini}} = \frac{1}{\frac{1}{S_j} + \frac{1}{S_s}} = S_j \tag{4}$$

$$M_R = \min(M_{j,R}; M_{s,R}) \tag{5}$$

where

$S_{\text{ini}} =$ initial stiffness of the integrated spring,
$M_R =$ ultimate moment capacity of the integrated spring.

The plastic-hinge model described above is accurate enough for the evaluation of the ultimate load but not accurate enough for tracing the frame load deflection characteristic. This is mostly because of member finite plastic zones and joint gradual yielding. The improvements made to the conventional hinge model resulted in the development of refined plastic hinge methods of analysis in which the combined effects of gradual stiffness degradation of members and joints are addressed.

3.2. Two surface stiffness degradation refined plastic hinge model

In the second order analysis using the two-surface stiffness degradation model the frame superelement is represented by an inelastic line element with one spring at each end that reproduces the joint flexibility and strength. The joint characteristic shown in Fig. 3(a) has to be replaced by the one that captures the continuous stiffness degradation effect. Among all different characteristics proposed, a three-parameter power model seems to be the most suitable [8]. Adopting this model, the spring tangent stiffness S_{jT} depends upon the actual state of moment redistribution process in the structural system and is calculated as follows:

$$S_{jT} = S_{j,\text{ini}}\left[1 - \left(\frac{M}{M_{j,R}}\right)^{n_j}\right]^{(n_j+1)/n_j} \tag{6}$$

where $n_j =$ shape factor of the joint characteristic.

The second order analysis with such an improved joint stiffness degradation model is accurate enough for frames with semi-rigid partial strength joints, provided that the joint ultimate moment capacity is placed below that corresponding to the onset of yielding in member cross sections. For frames with semi-rigid full strength joints, an important role is played not only by the effect of joint gradual stiffness degradation but also by the material yielding localized in plastic zones spreading from the member end sections. In this case, the rigid–plastic characteristic shown in Fig. 3(b) needs to be refined in order to account for the reduction in member stiffness resulting from permanent deformations integrated over the plastic zones.

In order to approximately capture the effect of permanent deformations, the discretisation scheme has to locate the mesh nodes at the points of maximum bending moments and furthermore all the member distributed loads need to be lumped at the nodes of the discretised structure. Since the plastic zones in the discretised frame model are developed at element ends and they propagate from the nodal points towards the element midsections, the structural behaviour is modelled within the least constrained moment redistribution process.

The inelastic behaviour of members and joints is to be captured at the element tangent stiffness relationship, see Eq. (7). The incremental equilibrium equation of the superelement i–k in the two surface stiffness degradation model can be expressed in local co-rotational coordinates as follows [9]:

$$\begin{bmatrix} \Delta M_{ik} \\ \Delta M_{ki} \\ \Delta P \end{bmatrix} = \begin{bmatrix} \frac{EI}{\xi L} s_{T,ii} & \frac{EI}{\xi L} s_{T,ik} & 0 \\ 0 & \frac{EI}{\xi L} s_{T,kk} & 0 \\ 0 & 0 & EA \end{bmatrix} \begin{bmatrix} \Delta\theta_i \\ \Delta\theta_k \\ \Delta\varepsilon \end{bmatrix} \tag{7}$$

in which:

$$s_{T,ii} = s_{mT,ii}^{ep} + \frac{EI}{S_{jT,k} L} \left(s_{mT,ii}^{ep} s_{mT,kk}^{ep} - {s_{mT,ik}^{ep}}^2\right) \tag{8}$$

$$s_{T,kk} = s_{mT,kk}^{ep} + \frac{EI}{S_{jT,i} L} \left(s_{mT,ii}^{ep} s_{mT,kk}^{ep} - {s_{mT,ik}^{ep}}^2\right) \tag{9}$$

$$\xi = \frac{1}{S_{jT,i} S_{jT,k}} \left(\frac{EI}{L}\right)^2 \left[\left(s_{mT,ii}^{ep} + \frac{S_{jT,i} L}{EI}\right) \times \left(s_{mT,kk}^{ep} + \frac{S_{jT,k} L}{EI}\right) - {s_{mT,ik}^{ep}}^2\right] \tag{10}$$

and

$\Delta P, \Delta M_{ik}, \Delta M_{ki} =$ axial force increment, and bending moment increments at the beginning and at the end of the element i–k, respectively,

$\Delta\varepsilon, \Delta\theta_i, \Delta\theta_k =$ axial strain increment, and rotation increments at the beginning and at the end of the element i–k, respectively,

$E =$ elasticity modulus,

$L =$ length of the element,

$I =$ second moment of area of the member section bent in the plane of structure,

$s_{mT,ii}^{ep}, s_{mT,kk}^{ep}, s_{mT,ik}^{ep} =$ member tangent inelastic direct stiffness coefficients at the end i and k, respectively, and tangent inelastic cross stiffness coefficient,

$S_{jT,i}, S_{jT,k} =$ joint tangent inelastic stiffness at the end i and k, respectively.

The member inelastic stiffness coefficients are calculated as follows:

$$s_{mT,ii}^{ep} = \rho_i s_{m,ii}^{e} \left[1 - (1 - \rho_k) \frac{{s_{m,ik}^{e}}^2}{s_{m,ii}^{e} s_{m,kk}^{e}}\right] \tag{11}$$

$$s_{mT,kk}^{ep} = \rho_k s_{m,kk}^{e} \left[1 - (1 - \rho_k) \frac{{s_{m,ik}^{e}}^2}{s_{m,ii}^{e} s_{m,kk}^{e}}\right] \tag{12}$$

$$s_{mT,ik}^{ep} = \rho_i \rho_k s_{m,ik}^{e} \tag{13}$$

where

$s_{m,ii}^{e}, s_{m,kk}^{e}, s_{m,ik}^{e} =$ member elastic direct stiffness coefficients at the end i and k, respectively, and elastic cross stiffness coefficient,

$\rho_i, \rho_k =$ scalar parameters that account for the effect of plastic zones on the member flexural stiffness at the ends i and k, respectively.

The scalar parameters ρ_i, ρ_k are the values of a stiffness degradation function ρ evaluated for the force states corresponding to the member ends i and k, respectively. The function ρ describes the reduction in the member flexural stiffness. The current value of this stiffness in the incremental analysis is dependent on the location of the force state point on the plane of axial force and bending moment. The force state point moves from the origin of the force coordinate system towards the initial stiffness degradation surface $F_e(M, N)$ (first yield load surface for which $\rho = 1$), and then towards the ultimate limit state surface $F_p(M, N)$ (limit load surface for which $\rho = 0$), as the bending moment redistribution process monitored in the incremental analysis progresses. The gradual reduction in the member inelastic stiffness is simulated by applying two scalar parameters, each allocated for a particular end of the member. Terms of the element stiffness matrix associated with the P–δ effect of flexural deflections are then multiplied by the expressions involving above parameters, see Eqs. (11)–(13). Depending on the current stage of moment redistribution process, the parameters change their values from unity (elastic range) to zero (plastic hinge formation). By choosing an appropriate stiffness degradation function, the effect of plastic zones can be effectively accounted for in the moment redistribution process. For details, refer to Liew et al. [9,10].

The formulation described above has been implemented in the computer program PHINGE and combined with the simple incremental method as a numerical technique for solving nonlinear problems (without an iterative search for the equilibrium configuration at each incremental stage of analysis). A copy of the program is an attachment to the publication edited by Chen and Toma [11] and hence disseminated among practicing structural engineers, academic staff and research students.

3.3. Spring-in-series refined plastic hinge model

In the second order analysis using the spring-in-series model the frame superelement is represented by an elastic

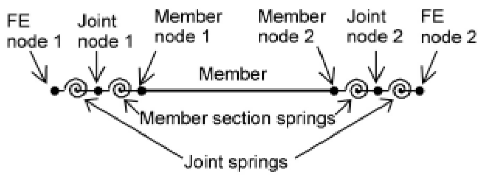

Fig. 4. Superelement used in spring-in-series refined plastic hinge method.

line element with two springs at each member end that reproduce the inelastic behaviour of end joints and the member itself. The superelement is shown in Fig. 4 [12]. Unlike in the conventional plastic hinge analysis, the moment–rotation spring characteristics have to represent the effect of gradual yielding. The plastic zone deformations can be integrated at each load increment and lumped at section springs at each end of the superelement. As a result, the spring characteristic is curvilinear, representing an increasing amount of permanent deformation in course of increasing the bending moment. When the effect of strain hardening is ignored, the curvilinear characteristic approaches the horizontal line representing the moment resistance of the plastic hinge as shown on the idealized rigid–plastic diagram in Fig. 3(b). Chan and Chui [12] have shown that the tangent stiffness of the spring representing the effect of plastic zones spreading from one member end of an I-section may be represented by the tangent stiffness that is function of the current force state:

$$S_{sT} = \frac{6EI}{L}\frac{M_{s,R} - M}{M - M_{s,e}} \quad \text{for } M - M_{s,e} \geq 0, M \geq 0 \qquad (14)$$

where

$M_{s,R} =$ section plastic moment resistance as in Eq. (5),
$M_{s,e} =$ section elastic moment resistance in the presence of axial force.

In the computer implementation, two springs at each end of the superelement can be integrated into one spring. The tangent stiffness of the integrated spring can be evaluated according to Eq. (4):

$$S_T = \frac{1}{\frac{1}{S_{jT}} + \frac{1}{S_{sT}}} \qquad (15)$$

and the moment resistance according to Eq. (5).

The element stiffness matrix in local co-rotational coordinates may be then expressed in the form of Eq. (7) with the member inelastic tangent stiffness coefficients replaced by their elastic counterparts [see Eqs. (8)–(10)]. This implies that line elements remain elastic while the entire plastic zone permanent deformations are lumped in the superelement section springs.

The refined plastic hinge analysis based on the integrated springs has a similar complexity as the one described in the previous subsection. It has to be emphasized that the tangent stiffnesses in Eq. (15) evaluated for both the section and the joint springs need to be recalculated at each step of the incremental analysis. The method has been implemented by Chan and Chui [12] in their computer program GMNAF that uses an incremental-iterative method as a numerical technique for tracing the prelimit and postlimit branches of the load deflection response of frame systems.

4. Methods of advanced analysis

In its simplest meaning, advanced analysis indicates an incremental second order analysis that can sufficiently capture the limit state strength and stability of the imperfect (real) structural system so that separate member and joint capacity checks encompassed by present code requirements are not needed [13]. The provisions for advanced analysis in recent design codes postulate that if all the significant planar behavioural effects are modelled properly in the analysis of plane frames sufficiently restrained in the out-of-plane direction, checking of conventional beam–column equations is not required, see AS 4100 [14] (also Eurocode 3: part 1.1). In light of the statements mentioned above, any refined plastic-hinge method of analysis allowing for the lumped effects of gradual stiffness degradation of joints and distributed plasticity of member sections can serve as advanced analysis provided that the effect of imperfections on the stability behaviour of structural members and the system as a whole is taken care of.

4.1. North American approach

Provisions for advanced analysis and limit state design were reported by Chen and Kim [13] and harmonized with the North American AISC LRFD Specifications [15]. In advanced analysis adopted by Chen and Kim [14], the refined plastic hinge method described in Section 3.2 was used. In order to facilitate advanced analysis, the effect of residual stresses on the behaviour of elements subjected to compression was taken care of by replacing the elasticity modulus E in Eq. (7) by the tangent modulus E_T. Its value in the inelastic region was calculated from the stress–strain diagram of a hypothetically ideal member that reproduced the stability behaviour according to the Shanley theory. The Shanley inelastic compressive strength conformed to the standard CRC buckling curve. It is meaningful to note that in this approach the same value of the tangent modulus was used to determine the member reduced rigidities $EI_r = E_T I$ in flexure and $EA_r = E_T A$ in axial compression or tension. The effect of geometrical imperfection was taken care of by three methods of modelling:

1. Explicit imperfection modelling.
2. Equivalent notional load modelling.
3. Reduced elasticity modulus modelling.

In method 1, geometrical imperfections representing the member initial bow and the frame initial sway are directly used in advanced analysis. In method 2, geometrical imperfections are replaced by a set of equivalent forces that in advanced analysis are to be added to externally applied loads. Finally, method 3 accounts for the effect of geometrical imperfections in an implicit way. In this method, the elasticity modulus of compression members is not treated as a material constant but rather a substitute elasticity modulus, referred to hereafter as the elastic buckling modulus reproducing the behaviour of slender members through member's elastic critical load formula. It gives:

$$E_b = \left(\frac{\pi}{\lambda}\right)^2 \frac{A}{P_b} \tag{16}$$

where

$\lambda =$ member slenderness,
$P_b =$ buckling load of slender members obtained from tests,
$E_b =$ elastic buckling modulus corresponding to P_b.

The buckling modulus may be treated as a random variable. AISC-LRFD Specification [15] gives statistically evaluated partial safety factors that can relate fractals of the buckling modulus distribution to the material constant E. The characteristic value of the buckling load of slender members is, according to AISC-LRFD Specification [15], given by:

$$P_b = \frac{0.877}{\left(\sqrt{\frac{P_y}{P_{cr}}}\right)^2} P_y \tag{17}$$

where

$P_y = Af_y$ squash load,
$P_{cr} = A\sigma_{cr}$ elastic critical load,
$f_y, \sigma_{cr} =$ characteristic (nominal) strength and elastic critical stress.

From Eq. (17), the partial safety factor $\gamma_c = 1.14$ is to be applied to the elasticity modulus E for the evaluation of the characteristic value of buckling modulus E_c. The design criterion of compression members is in AISC-LRFD Specification [15] of the following format:

$$P \leq \phi P_b \tag{18}$$

with the resistance factor $\phi = 0.85$. The partial safety factor γ_d is applied to the characteristic value of the elastic buckling modulus E_c in order to evaluate the design value of buckling modulus E_d. From Eq. (18) its value is $(1/0.85) = 1.17$. The modulus E_d can now be related to the material constant through the following equation:

$$E_d = \frac{E_c}{\gamma_d} = \frac{E}{\gamma_c\gamma_d} = \frac{E}{1.14 \cdot 1.17} = \frac{E}{1.33} = \frac{E}{\gamma_{cr}}. \tag{19}$$

Eq. (19) can be written as $E_d = 0.75E$. The resultant design value of the buckling modulus agrees with that used in the national code PN-90/B-03200 [16]. The same partial safety factor $\gamma_{cr} = 1.33$ is therefore used in both the codes for design of slender compression members.

Advanced analysis with all three methods of imperfection modelling was implemented in the computer program PAAP by Chen and Kim [13] and attached to the above publication. Through a validation exercise, the values of substitute geometric imperfections and notional load factors were selected for methods 1 and 2, respectively. For method 3, the value of $0.85E$ was recommended for compression members in both braced and unbraced frames. It is to be noted that the adopted value is larger than the design value used to represent the buckling design strength of slender members [see Eq. (19)].

4.2. *European approach*

In the recent European codes it is recommended that the combined effect of imperfections be modelled by using the concept of equivalent geometric imperfections or by using the system of equivalent loads (e.g. Eurocode 3: part 1.1). Both the concepts combine the effect of all the imperfections into one design factor represented by geometric imperfections (bow and/or sway) or incorporated as their $P-\Delta$ equivalents to the set of notional loads. They are similar to the concepts used in methods 1 and 2 in the North American approach. In both the cases it is required that the direction of imperfections in structural members and/or at frame stories, or the direction of loads representing the effect of imperfections, need to be carefully chosen in order to capture their most unfavourable effect on the structural stiffness degradation. Since the determination of the said directions requires the prediction of the buckling mode corresponding to the lowest bifurcation load, the methods of equivalent geometric imperfections and equivalent loads are somewhat inconvenient in practical applications. It is furthermore notable that the buckling mode is dependent on the axial force distribution and as such it varies for different load combinations. The influence of imperfections therefore needs the determination of buckling modes for all the load combinations included in design prior to the performance of advanced analysis.

Research that has been initiated by Chen and Kim [13] for the development of method 3 can also be applied in conjunction with recent European codes. The method 3 is further being developed and referred hereafter as the CSD method (Continuous Stiffness Degradation method). If this method is combined with the spring-in-series model it can effectively account for the effects of distributed plasticity and imperfections in advanced analysis. The method is being incorporated in the computer program LILAN-N [17].

Recent research by Byfield and Nethercot [18], Kemp et al. [19] and Byfield et al. [20] has emphasized the need to consider the influence of strain hardening on inelastic stability behaviour of steel frames. This effect becomes more important in advanced analysis since it tends to treat instability in a more rigorous way than the case when design is based on the first plastic hinge concept. The strain hardening effect therefore deserves more attention to be paid in modelling of the inelastic behaviour of both members and joints. The following explains how the effect of kinematic hardening may be incorporated into advanced analysis based on the CSD method.

The kinematic hardening effect in the joint behaviour can be included by using the moment–rotation characteristic of the joint spring suggested by Gizejowski et al. [21]:

$$\frac{M}{M_{j,R}} = \left[\left(1+\alpha_{jh}\frac{\phi}{\phi_d}\right)^{-n_j} + \left(\frac{\phi}{\phi_d}\right)^{-n_j}\right]^{-\frac{1}{n_j}} \tag{20}$$

where

$\phi_d = M_{j,R}/S_{j,\text{ini}}$ rotation associated with yielding of the joint in a bilinear joint characteristic (see Fig. 3(a)),
$\alpha_{jh} =$ kinematic hardening factor that relates the hardening modulus $S_{j,h}$ to the initial modulus $S_{j,\text{ini}}$.

Based on the current design specifications (e.g. Eurocode 3: part 1.8) and joint behaviour experimental data base (e.g. [22]), one can propose the shape factor n_j and the hardening modulus factor α_{jh} from Table 1. It is worth noting that Eq. (20) reduces to the one proposed by Chen and Kishi [8] if the effect of kinematic hardening is neglected ($\alpha_{jh} = 0$).

Table 1
Parameters for joint spring moment–rotation characteristic

Joint type	Factors	
	n_j	α_{jh}
End plate	1.60	0.045
Flange angles	1.50	0.035

The effect of residual stresses on the development of plastic zones in bending and strain hardening on the moment–curvature relationship can be accounted by specifying the shape parameter of the section spring moment–rotation characteristic. This characteristic can be described by Eq. (20), i.e. by the same equation as the joint characteristic but using different model parameters. Parameters $M_{s,R}$, $S_{s,\text{ini}}$, α_{sh}, n_s need to be used instead of the parameters $M_{j,R}$, $S_{j,\text{ini}}$, α_{jh}, n_j. Based on Eq. (14) for the prediction of the combined effects of residual stresses and plastic zones, the calibration of the n_s parameter requires the values of compressive residual stress to be taken from ECCS Publication [3]. The summary of calibration is given in Table 2 [17]. The value of $60EI/L$ is suggested for the initial stiffness of the member section spring. Based on Byfield et al. [20], the hardening modulus factor can be taken as 0.0033 if relating the hardening modulus to the initial stiffness $60EI/L$.

The element stiffness matrix in advanced analysis according to CSD method is of the same form as in the second order spring-in-series model (see Section 3.3) but an allowance has to be made for the effect of imperfections on the member behaviour under axial compression or tension.

Table 2
Shape factors for section spring moment–rotation characteristic

Factor	Rolled I-sections		Welded I-sections
	$D/B \le 1.2$	$D/B > 1.2$	
n_s	1.25	1.60	0.75

This is accounted by the substitution of the rigidities EI and EA in the member stiffness matrix by their reduced quantities:

$$EI_r = \tau E_d I \tag{21}$$

$$EA_r = \kappa EA \tag{22}$$

where

$\tau =$ flexural stiffness degradation function,
$\kappa =$ axial stiffness degradation function.

The member reduced flexural and axial stiffnesses are calibrated in such a way that they reproduce globally the combined effect of geometric imperfections and residual stresses on the stability behaviour of individual elements. Barszcz and Gizejowski [23] developed a procedure for the calibration of degradation functions that avoids an explicit use of substitute geometric imperfections or notional loads.

The instability model based on the inelastic bifurcation theory is used to evaluate the degradation function for the flexural stiffness. The generalized stress–strain relationship of a hypothetically perfect element is evaluated [see curve (a) in Fig. 5]. The following notation holds: $\varphi = P/Af_d$ is the buckling reduction factor, $\overline{\lambda} = \sqrt{\gamma_{\text{cr}} A f_d / P_{\text{cr}}}$ is the relative slenderness ratio, $f_d = f_y/\gamma_m$ is the steel design strength, and finally γ_m is the strength partial safety factor. The above relationship reproduces the buckling strengths through the buckling curve of the imperfect compression member. Each point on the buckling curve represents the strength of an imperfect member evaluated for its hypothetical perfect counterpart with use of the Shanley bifurcation stability criterion. The expression adopted for the σ–ε diagram can be written down as follows:

$$\frac{\sigma}{f_d} = \left[\left(1+\alpha_{bh}\frac{\varepsilon}{\varepsilon_d}\right)^{-n_b} + \left(\frac{1}{\gamma_{\text{cr}}}\frac{\varepsilon}{\varepsilon_d}\right)^{-n_b}\right]^{-\frac{1}{n_b}} \tag{23}$$

where

$\sigma = P/A$ generalized stress produced by the compressive axial force P,
$\varepsilon = \Delta/L$ generalized strain calculated on the basis of strut shortening Δ,
$\alpha_{bh} = E_h/E$ strain hardening modulus factor relating the hardening modulus E_h to the modulus of elasticity E,
$\varepsilon_d = f_d/E$ strain at the attainment of steel design strength in a bilinear σ–ε relationship,
$n_b =$ shape factor.

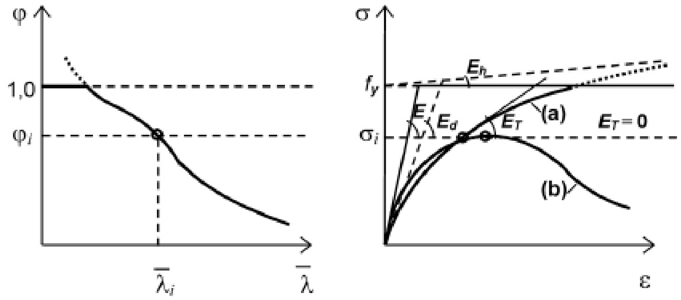

Fig. 5. Stress–strain relationships reflecting the member stability behaviour in the CSD method.

The tangent modulus is evaluated from the following equation:

$$\tau = \frac{1}{E_d}\frac{\mathrm{d}\sigma}{\mathrm{d}\varepsilon} = \frac{\alpha_{bh}\gamma_{\mathrm{cr}}\left(\frac{1}{\gamma_{\mathrm{cr}}}\frac{\varepsilon}{\varepsilon_d}\right)^{n_b+1} + \left(1+\alpha_{bh}\frac{\varepsilon}{\varepsilon_d}\right)^{n_b+1}}{\left[\left(\frac{1}{\gamma_{\mathrm{cr}}}\frac{\varepsilon}{\varepsilon_d}\right)^{n_b} + \left(1+\alpha_{bh}\frac{\varepsilon}{\varepsilon_d}\right)^{n_s}\right]^{\frac{(1+n_b)}{n_b}}}. \tag{24}$$

The buckling design strength therefore is to be evaluated as:

$$P_b = \frac{\pi^2 E_T I}{\lambda^2} = \frac{\pi^2 \tau E_d I}{\lambda^2}. \tag{25}$$

Parameters α_{bh}, γ_{cr} can be obtained from current codes if they are specified there explicitly. Otherwise they can be calibrated together with the shape factor n_b. The calibration requires Eq. (25) to give a close approximation of buckling strengths to those evaluated from the buckling curve considered in calibration. The calibration was performed by Barszcz and Gizejowski [23] in order to reproduce buckling curves from the code PN-90/B-03200 [16].

The instability model based on the limit point on the generalized stress–strain relationship is used to evaluate the degradation function for the axial stiffness. The generalized stress–strain relationship of a hypothetically perfect element consists of the ascending branch, limit point and descending branch [see curve (b) in Fig. 5]. It reproduces the behaviour of the imperfect compression member through the divergence type of instability criterion and as such it can be expressed as follows:

$$\frac{\sigma}{f_d} = \left[\left(1+\alpha_{bh}\frac{\varepsilon}{\varepsilon_d}\right)^{-n_b} + \left(\frac{\varepsilon}{\varepsilon_d}\right)^{-n_b} + \left(\frac{1}{\bar{\lambda}^2}\right)^{-n_b} + \left(\frac{1}{\pi\bar{\lambda}\sqrt{\beta\frac{\varepsilon}{\varepsilon_d}}}\right)^{-n_b}\right]^{-\frac{1}{n_b}} \tag{26}$$

where $\bar{\lambda} = \sqrt{\gamma_{\mathrm{cr}} A f_d / P_{\mathrm{cr}}}$ relative slenderness ratio.

The tangent modulus is evaluated from the following equation:

$$\kappa = \frac{1}{E}\frac{\mathrm{d}\sigma}{\mathrm{d}\varepsilon} = \left[\left(1+\alpha_{bh}\frac{\varepsilon}{\varepsilon_d}\right)^{-n_b} + \left(\frac{\varepsilon}{\varepsilon_d}\right)^{-n_b} + \left(\frac{1}{\bar{\lambda}^2}\right)^{-n_b} + \left(\frac{1}{\pi\bar{\lambda}\sqrt{\beta\frac{\varepsilon}{\varepsilon_d}}}\right)^{-n_b}\right]^{-\frac{1+n_b}{n_b}} \times \left[\alpha_{bh}\left(1+\alpha_{bh}\frac{\varepsilon}{\varepsilon_d}\right)^{-(1+n_b)} + \left(\frac{\varepsilon}{\varepsilon_d}\right)^{-(1+n_b)} - \frac{1}{2\pi\bar{\lambda}\sqrt{\beta}\left(\sqrt{\frac{\varepsilon}{\varepsilon_d}}\right)^3}\left(\frac{1}{\pi\bar{\lambda}\sqrt{\beta\frac{\varepsilon}{\varepsilon_d}}}\right)^{-(1+n_b)}\right]. \tag{27}$$

Parameters α_{bh}, γ_{cr} can be obtained from current codes if they are specified there explicitly. Otherwise they can be calibrated together with the shape factor n_b and the section parameter β. The calibration requires that limit points on the generalized σ–ε characteristic given by Eq. (26) are in close approximation to buckling strengths of the buckling curve considered in calibration. It can be shown that buckling curves constructed from the maxima of function given by Eq. (26) agree, for parameters α_{bh}, γ_{cr}, n_b that are the same as in the bifurcation model and for an optimally calibrated value of β, with the buckling curve adopted for calibration. Parameters harmonized with the national steel code PN-90/B-03200 [16] are given by Barszcz and Gizejowski [23].

5. Design example

An illustrative example of the four-bay and six-story frame is considered [24]. The frame geometry and loading is given in Fig. 6 and the beam-to-column joint detail in Fig. 7. Steel grade Fe 360 is used. Six characteristics of semi-rigid partial strength joints are identified. They account for the deformability of beam-to-column connections and the column web panel zone effect. The properties of joints are listed in Table 3.

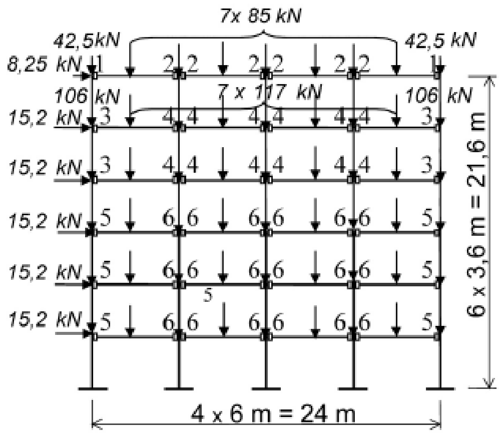

Fig. 6. Frame geometry and loading.

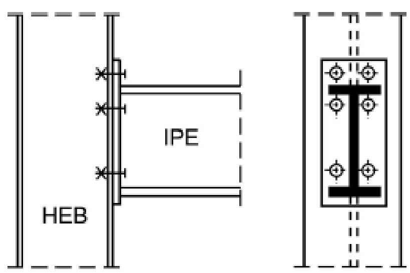

Fig. 7. Joint construction detail.

Table 3
Properties of semi-rigid joints

Joint type	Ultimate moment (kN m)	Initial stiffness (kN m/rad)
1 – IPE 270 to HEB 180	65	16 100
2 – IPE 270 to HEB 180	80	54 500
3 – IPE 330 to HEB 180	83	25 700
4 – IPE 330 to HEB 180	105	84 500
5 – IPE 330 to HEB 240	99	31 700
6 – IPE 330 to HEB 240	112	79 300

The elastic design process was carried out automatically with use of the commercial software ROBOT Millennium. The following options were selected in the design: elastic second order analysis and Polish steelwork design code, PN-90/B-03200 [16]. The design output is verified herein by the inelastic design with use of other two computer programs, PHINGE-R and LILAN-N, that are capable of advanced analysis and design. The same assumptions for modelling are used in analyses performed by both the computer programs. The computer program PHINGE-R is the implementation of the North American approach that has been harmonized by Gizejowski [25] with the national steel design code PN-90/B-03200 [16]. The program LILAN-N is being developed by the authors [17]. Since the code PHINGE-R does not account for hardening effects, these effects are also omitted in running the program LILAN-N. The latter computer program allows for different analyses as well as different ways of capturing material yielding and inelastic joint deformations. In order to compare the results of different assumptions for material yielding, program PHINGE-R was run with use of the refined plastic hinge method of analysis while the option of conventional plastic-hinge model for the section spring is assumed in LILAN-N. For comparison purposes, the computer code LILAN-N is used with options as follows: FO-BJ (First Order analysis and Bilinear Joint characteristics), SO-BJ (Second Order analysis and Bilinear Joint characteristics) and SO-CJ (Second Order analysis and Curvilinear Joint characteristics corresponding to the joint characteristic of Chen and Kishi [8] with the shape factor equal to 1.5). The frame load deflection characteristics are shown in Fig. 8.

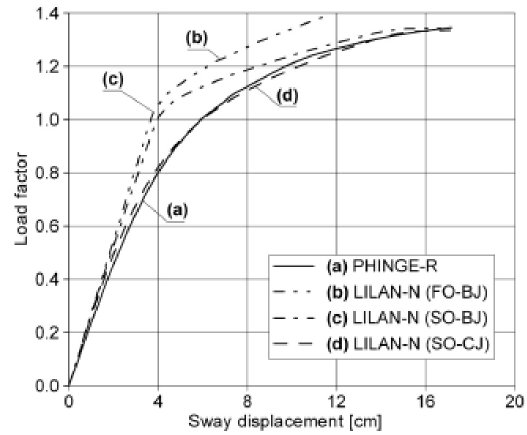

Fig. 8. Frame load deflection characteristics.

The results of the ultimate limit state analyses are summarized in Table 4. From the results in Table 4, it is clear that all the analyses considered give a close estimation of the frame ultimate load but different load deflection characteristics and displacements at the frame ultimate state. The first order analysis according to LILAN-N (option FO-BJ) cannot be used for the reasonable estimation of the frame equilibrium path and deflections in the frame ultimate state calculations. The second order analysis according to LILAN-N with plasticity effects lumped in member sections and in beam-to-column joints (option SO-BJ) gives a better estimation of the frame ultimate load as well as an improved estimation of the frame load deflection characteristic. The second order analysis according to LILAN-N with the inclusion of gradual stiffness degradation in joints (option SO-CJ) gives practically the same results as advanced analysis by PHINGE-R in which the effect of member plastic zone deformations is accounted for. The results prove an intuitive statement that for frames with semi-rigid partial strength joints the effect of member distributed plasticity is negligible and may be disregarded in design when using advanced analysis, in contrast to frames with semi-rigid full strength joints.

Table 4
Results from different inelastic analyses

Computer code (type of analysis)	Ultimate load factor	Sway deflections of the top story (cm)
PHINGE-R	1.346	17.1
LILAN-N (FO-BJ)	1.384	11.4
LILAN-N (SO-BJ)	1.344	16.7
LILAN-N (SO-CJ)	1.336	17.1

Advanced analyses show that the designed structure has the plastic load carrying reserve of approximately 30%. When redesigning with use of advanced analysis performed by PHINGE-R, a more economical solution can be obtained than using present design specifications. The application of advanced analysis allows for smaller floor beams to be adopted, namely IPE 300 instead of IPE 330.

6. Further development

The example presented in the previous section and a number of other simulations carried out with the use of advanced analysis software by Chen and Kim [13], Chan and Chui [12] and others, prove that design procedures in the present codes based on the component strength and stability criteria ensure neither economical solutions nor the consistent reliability level of designed structures. Use of advanced analysis leads to generally smaller member sizes due to benefit of the inelastic moment redistribution and in all the cases due to achieving more uniform values of the reliability index.

The development of design procedures based on advanced analysis is a key consideration for the future design codes. It is therefore postulated that methods of advanced analysis in a simple form based on methods and concepts taught at the university undergraduate level and yet consistent with the present limit states codes need to be developed. The concept based on the second order analysis and the spring-in-series model to account for inelastic deformations of frame structural components is the most promising one provided that it is combined in the computer implementation with the simple incremental solution procedure. The CSD method seems to be the most useful as far as the effects of imperfections are considered in advanced analysis. Investigations are undertaken to develop the CSD method further in order to conveniently account for the hardening effect and the influence of imperfections in structural components and in the structural system as a whole. Model parameters of the CSD method need to be calibrated in the European approach in order to reflect the provisions of Eurocode 3: part 1.1 so that they can be utilized in a simple version of advanced analysis. The computer code LILAN-N is being developed in order to implement the method and offer it for use by students and structural engineers. In the information society witnessed nowadays, structural engineers seem to accept the method of advanced analysis as a design tool in their day-to-day practice.

7. Conclusions

The paper summarizes the state-of-the-art and trends in the development of methods of analysis used in the design of semi-continuous frame systems. Present design codes recommend different methods of analysis for use in structural assessment, such as first-order, simplified second-order and second-order methods, depending on the design situation and the availability of computer software. It creates room for arbitrary judgment of the design situation. An arbitrary choice of analysis methods may be the cause for the erection of structural systems characterized by a greater variability of safety level. In the era of rapid progress in limit-state assessment based on the required target reliability indexes, there are still considerable inconsistencies resulting from discrepancies in the modelling of structural loads and their combinations, and in the modelling of structural response. The paper has discussed advanced methods of structural analysis towards the assessment of structural response. Use of advanced analysis can benefit the design process by making it more economical and more transparent, and linking it with the real behaviour of the structure at ultimate state.

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