

# Single-electron states near a current-carrying core

M. Masale\*

*Department of Physics, University of Botswana, Private Bag 0022, Gaborone, Botswana*

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## Abstract

The energy spectrum of an electron confined near a current-carrying core is obtained as a function of the azimuthal applied magnetic field within the effective-mass approximation. The double degeneracy of the non-zero electron's axial wave number ( $k_z$ ) states is lifted by the current-induced magnetic field while that of the non-zero azimuthal quantum number ( $m$ ) states is preserved. A further analysis is the evaluations of the oscillator strengths for optical transitions involving the lowest-order pair of the electron's energy subbands within the dipole approximation. The radiation field is taken as that of elliptically polarized light incident along the core axis. In this polarization and within the dipole approximation, the allowed transitions are only those governed by the following specific selection rules. The azimuthal quantum numbers of the initial and final states must differ by unity while the electron's axial wave number is conserved. The azimuthal magnetic field is also found to lift the multiple degeneracies of the  $k_z \neq 0$  interaction integrals as well as those of the oscillator strengths for optical transitions.

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## 1. Introduction

The interest in the dynamics of electrons under the influence of a static uniform magnetic field grew even more rapidly with the realization of technologies to fabricate nanostructures [1,2]. This renewed interest was also, in part, due to potential device applications, which rely on the reduced phase space of the charge carriers [3]. It needs to be emphasized that the orientation of the applied magnetic field relative to the heterointerfaces of low-dimensional systems is crucial in that it

determines the broad character of the confined charge carriers. Perhaps as a simplification, the usual practice is to consider the application of a magnetic field either parallel or perpendicular to the heterointerfaces of plano-surface low-dimensional systems. In recent years, systems in which the applied magnetic field is spatially inhomogeneous have received a great deal of attention [4]. Systems studied include nanostructures with plano-heterointerfaces [5–11], semiconductor rings [12], magnetic rings [13] as well as magnetic antidots [14]. A magnetic field with non-zero gradient can be created by simply fashioning the core of the electromagnet, for example, to have a conical end-face. Some relatively recent techniques

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\*Tel.: +267-3552940; fax: +267-585097.

E-mail address: masalem@mopipi.ub.bw (M. Masale).

involve integrating semiconductors with ferromagnetic [15] or superconducting materials [16]. There has also been some limited discussion of the influence of an azimuthal inhomogeneous magnetic field on the charge carriers based on a semiclassical formalism [17]. The uniform-gradient azimuthally directed magnetic field there is thought to be induced by a current passed along the length of a core, which in practice might be a superconductor. Clearly, the dynamics of electrons under the influence of a spatially inhomogeneous magnetic can no longer be characterized uniquely, for example, in terms of the cyclotron radius or cyclotron frequency. In any case the electron's orbits are not closed for a wide range of the relevant parameters [17] but are found to be predominantly snake-like in character [9–11]. In other developments, experimental techniques for the control of the specific number of electrons that can be “injected” into a nanosystem are now well established [12]. A significant implication of this development is that, neglecting electron–electron interactions in the evaluations of the other quantum properties of the charge carriers is no longer, necessarily, a gross simplification.

The aim of the investigations undertaken here is to map out the energy spectrum of an electron confined near a current-carrying core as a function of the current-induced azimuthal magnetic field. The novel feature of the problem posed here is the orientation of the spatially inhomogeneous magnetic field relative to the surface of the heterostructure. An external magnetic field, no matter how weak, can have a significant influence on the general properties of charge carriers in low-dimensional systems. Now, a current-induced magnetic field is essentially an integral part of electrical devices. It is important, therefore, to evaluate the influence of such a field on the charge carriers. An understanding of the basic properties of electrons under the influence of an azimuthal magnetic field should be of value in the realization of technologies for potential device applications. As a contribution towards the understanding of properties of quantum electrons, having obtained the eigenfunctions, these are then employed in the calculations of the oscillator strengths for optical transitions.

This paper is organized as follows: the eigenvalue equation is derived in Section 2; the general formalism for calculating the oscillator strengths for optical transitions in systems with cylindrical symmetry is presented in Section 3; and the conclusions are contained in Section 4.

## 2. Energy eigenvalues

A brief description of the system studied here is as follows: a current  $I$  of uniform density  $J_{\text{ext}}$  is passed along the axis of a cylindrical core of radius  $R$ , possibly as a superconductor. The core is thought to be ensheathed with a perfect insulator, in essence constituting an infinite potential barrier. The purpose of the barrier is to prevent the leakage of current into the host material as well as to restrict the motion of the electrons strictly to the host material. For convenience, the spatial thickness of the potential barrier is assumed to be zero. Electrons are confined near the core, in part, by an electric potential, which is modelled as follows:

$$V(\rho) = \begin{cases} \infty & \text{for } \rho < R, \\ \frac{1}{2}\mu\omega_0^2(\rho^2 - R^2) & \text{for } \rho > R, \end{cases} \quad (1)$$

where  $\omega_0$  is the angular frequency of a classical simple harmonic oscillator of mass  $\mu$ , taken to be the same as the effective mass of an electron. The motion of the electrons, limited to the region outside the current-carrying core, is influenced by the spatially inhomogeneous current-induced azimuthal magnetic field. The vector potential of the external magnetic field,  $\mathbf{B} = \mu_0 I \hat{\phi} / (2\pi\rho)$ , is taken in the gauge

$$A_z = -\frac{1}{2}B_s R[1 + 2\ln(\rho/R)], \quad (2)$$

where  $B_s = \mu_0 I / (2\pi R)$  is the value of the magnetic field at the surface of the core. The solution of the Schrödinger equation is sought in the general form  $\psi = C_{m\ell} \exp(ik_z z) \exp(im\phi)\chi(\rho)$ ,

$$m = 0, \pm 1, \pm 2, \dots, \quad (3)$$

where  $C_{m\ell}$  is a normalization constant,  $k_z$  is the axial component of the electron's wave vector, and  $\ell$  and  $m$  are the radial and azimuthal quantum numbers, respectively. It is convenient to define the fictitious “cyclotron radius”,  $a_{\text{cs}} = (\hbar/eB_s)^{1/2}$ , and

the fictitious “cyclotron frequency”,  $\omega_{cs} = eB_s/\mu$ , in terms of the value of the magnetic field at the surface of the core. With the forms of the confining potentials,  $V(\rho)$  and  $A_z$ , the radial function  $\chi(\rho)$  is found to satisfy the following second-order linear differential equation:

$$\eta \frac{d}{d\eta} \left( \eta \frac{d\chi}{d\eta} \right) - \left\{ \frac{1}{4} m^2 + \frac{1}{4} [k_z^2 R^2 / f_{cs} - 4E_{mf} / \hbar \omega_{cs} - 4f_{cs} \omega_0^2 / \omega_{cs}^2 - 2k_z R + f_{cs}] \eta + \omega_0^2 \eta^2 / \omega_{cs}^2 - \frac{1}{4} \eta \ln(\eta / f_{cs}) \right\} \chi = 0, \quad (4)$$

where  $\eta = \rho^2 / 2a_{cs}^2$ ,  $f_{cs} = R^2 / 2a_{cs}^2$  and  $E_{mf}$  is the confinement part of the electron's total energy,  $E_{Tot}$ , given by

$$E_{Tot} = E_{mf} + \frac{\hbar^2 k_z^2}{2\mu}, \quad (5)$$

in which the second term is the axial kinetic energy. It is not possible, in particular, because of the logarithmic terms, to cast Eq. (4) into a

$$\beta = \frac{[k_z^2 R^2 / f_{cs} - 4E_{mf} / \hbar \omega_{cs} + (1 - 4\omega_0^2 / \omega_{cs}^2 + 2c_1 + d_1) f_{cs} - 2(1 + c_1) k_z R]}{[d_2 + 4\omega_0^2 / \omega_{cs}^2 + 2c_2(1 - k_z R / f_{cs})]^{1/2}}. \quad (11)$$

canonical form. However, solutions of Eq. (4) may be found in closed form if the logarithmic terms are replaced by their linear fitting forms as follows:

$$\ln x \approx c_1 + c_2 x \quad \text{and} \quad \ln^2 x \approx d_1 + d_2 x. \quad (6)$$

The constant factors  $c_i$  and  $d_i$  ( $i = 1$  or  $2$ ), which depend on the range of the values of  $x$  used, are simply read off the computer on using graphical software packages, for example, such as MSDOS-grapher. It is worth commenting that a much closer fit is found for the second ( $\ln^2 x$ ) of these terms. Note that the first of the above approximations leads to a form of the potential which corresponds to a uniform azimuthal magnetic field of magnitude:  $|B| = c_2 B_s$ . This result does not differ much from the crude average value  $\bar{B} = \frac{1}{2} B_s$  of the actual magnetic field, for regions very near the surface of the core, that is,  $\rho \approx R$ . It is important, for the validity of the subsequent

results, that the strength of the confining electric potential is suitably chosen so as to confine electrons to regions near the surface of the core. With the logarithmic terms now replaced by their approximate linear graphical fittings, the substitution

$$\chi = \zeta^{|m|/2} e^{-\zeta/2} \mathcal{F}, \quad (7)$$

where

$$\zeta = [d_2 + 4\omega_0^2 / \omega_{cs}^2 + 2c_2(1 - k_z R / f_{cs})]^{1/2} \eta, \quad (8)$$

leads to

$$\zeta \frac{d^2 \mathcal{F}}{d\zeta^2} + (b - \zeta) \frac{d\mathcal{F}}{d\zeta} - a\mathcal{F} = 0, \quad (9)$$

which is the canonical form of Kummer's equation for the confluent hypergeometric function. The solution that is bounded at  $\zeta = \infty$  is  $\mathcal{F} = U(a, b, \zeta)$ , the parameters  $a$  and  $b$  of which are given by

$$a = \frac{1}{2} + \frac{1}{2}|m| + \frac{1}{4}\beta \quad \text{and} \quad b = |m| + 1, \quad (10)$$

in which

The application of the standard boundary condition, that of continuity of the wave function at the surface of the core  $\rho = R$ , leads to the following dispersion relationship for the determination of the electron's energy eigenvalues:

$$U(a, b, \zeta_R) = 0, \quad (12)$$

where  $\zeta_R$  is given by Eq. (8) but with the replacement,  $\rho = R$ . Now, the value of  $B_s$  can easily be computed from the given values of  $I$  and  $R$ , hence the other relevant quantities, for example,  $a_{cs}$  and  $\omega_{cs}$  can be determined. Mapping out the single-electron energy spectrum as a function of the field is then a matter of the usual root-finding routine—fix  $f_{cs}$ ,  $|m|$  and  $k_z R$  and then search for the  $E_{mf}$  that satisfies the dispersion equation. The numerical value of the effective-mass of the electron is taken as roughly 0.067 times that of the free electron. It is convenient to define an index of the strength of the electric potential,  $\alpha$ , given by

$\alpha^2 = V(\sqrt{2R})/V_0$ , where  $V_0 \sim 190$  meV is the conduction offset between GaAs and  $\text{Al}_{0.25}\text{Ga}_{0.75}\text{As}$ . It is important to restrict  $\alpha$  to low values since the electron's energies are determined in the effective mass approximation. However, in view of the approximations of the logarithmic terms in Eq. (6),  $\alpha$  should also be such that the electrons are confined to a region within the vicinity of the core. Suitable values of  $\alpha$  can be estimated from the spread of the radial wave function [18]. The spatial extent of the wave function directly prescribes the appropriate range of the values of  $x$  which in turn leads to the determination of particular numerical values of the constants  $c_i$  and  $d_i$ . Alternatively,  $\alpha$  can be adjusted so that the radial wave function does not spread much beyond a predetermined range of  $\rho/R$  values. Choosing the latter approach, within the range  $1 \leq \rho/R \leq 5$ , very good fittings to the above mentioned logarithmic terms are obtained with the following constants:  $c_1 = -0.1122$ ,  $c_2 = 0.373$ ,  $d_1 = -0.849$  and  $d_2 = 0.688$ .

For ease of graphical presentation of the results, the variation of only the first two lowest-order— $m, \ell = 0, 1$  and  $1, 2$ —energy subbands with the surface-value of the magnetic field,  $B_s$ , is illustrated in Fig. 1. The variation of the higher  $\{m, \ell\}$  quantum number energy subbands with the field is very similar to those shown here and is therefore left out for brevity. The relevant parameters for the system considered are:  $R = 50 \text{ \AA}$ ,  $\alpha = 0.025$  and the pairs of  $\{m, k_z R\}$  values indicated there, also used to identify the different curves. To be more specific, the  $\{m, k_z R\}$  values are:  $\{0, 0.00\}$  for the thick solid curves,  $\{0, -0.10\}$  and  $\{0, 0.10\}$  for the dashed and the solid smooth curves, respectively. Further, the lowest group of the three curves at  $B_s = 0T$ , say, correspond to  $\ell = 1$  and the upper group of the three curves, at the same value of the field, correspond to  $\ell = 2$ . It is seen that the azimuthal magnetic field lifts the double degeneracy of the non-zero  $k_z$  energy subbands. The  $k_z > 0$  subbands initially decrease in energy and are characterized by minima in their variation with the magnetic field. For small fields, the Zeeman splitting is almost linear, however, for large fields all the energy subbands show a somewhat parabolic rise in  $B_s$ . The initial decrease

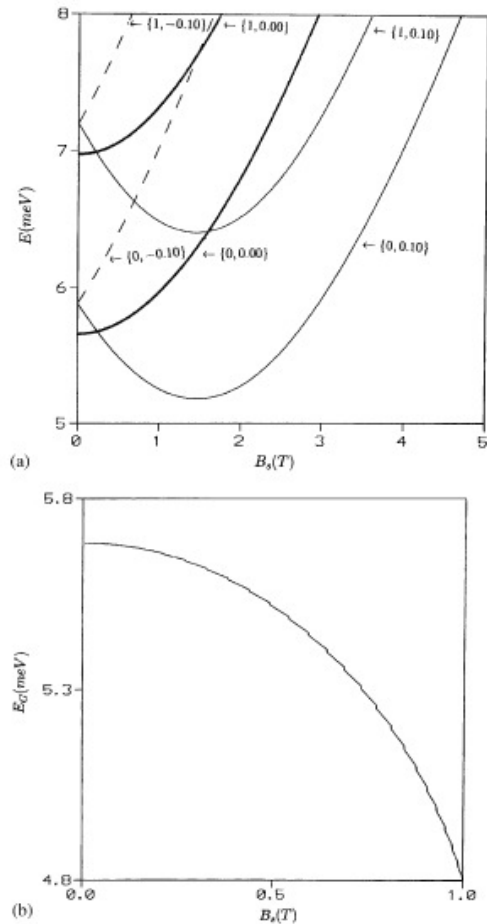


Fig. 1. (a) Some of the lowest-order single-electron energy subbands as functions of the surface-value of the current-induced magnetic field. The key relevant parameters used are  $R = 50 \text{ \AA}$ ,  $\alpha = 0.025$ ,  $\ell = 1, 2$  and the pair of  $\{m, k_z R\}$  shown there to identify the individual curves. These are  $\{0, 0.00\}$  for thick solid curves,  $\{1, -0.10\}$  and  $\{1, 0.10\}$  for the dashed and the smooth solid curves, respectively. Further,  $\ell = 1$  for the lowest group of the three curves at  $B_s = 0T$ , say, and  $\ell = 2$  for the upper group of the three curves. (b) The variation of the ground state ( $m = 0, \ell = 1$ ) energy  $E_G$  with  $B_s$ . The relevant parameters are:  $R = 50 \text{ \AA}$ ,  $\alpha = 0.025$  and the range  $0.00 \rightarrow 0.26$  of  $k_z R$  values, in steps of 0.01.



of the  $k_z > 0$  subbands with the increase of the field is due to the fact that in this case the two parts of the axial canonical momentum— $P_z$  and  $eA_z$ —are oppositely directed. For  $k_z < 0$ , the two parts of the momentum are additive, hence the electron's energy subbands increase monotonically with the increase of the field. The condition for obtaining minima in the variations of the  $k_z > 0$  energy subbands, based on a perturbative analysis, may be cast as follows:

$$\pi B_s \langle \rho^2 \rangle = \phi_0 k_z R, \quad (13)$$

where  $\phi_0 = h/e$  is the elementary flux quantum; the ratio of Planck's constant ( $h$ ) to the electronic charge ( $e$ ) and  $\langle \rho^2 \rangle$  is the quantum mechanical expectation value of the square of the radial distance. A somewhat paradoxical interpretation of Eq. (13) is that minima of the  $k_z > 0$  energy subbands occur whenever  $k_z R$  elementary flux quanta are enclosed within the electron's fictitious cyclotron radius,  $a_{cs}$ . Note that here, in contrast to the case of an axial applied magnetic field; see for example Masale [18]; the double degeneracy of the  $m \neq 0$  states is preserved. As a consequence of having a forbidden inner region for the motion of the electron ( $\rho < R$ ), the  $\{m = 0; k_z = 0\}$  subband is not always the ground state. In fact the Zeeman splitting of the  $k_z \neq 0$  states by the azimuthal field is such that as  $B_s$  increases, there arises a progression of ground-state energies corresponding to increasing values of the electron's axial wave number. As mentioned earlier, because of the spatial inhomogeneity of the applied magnetic field, the electron's orbits are not closed for a wide range of the relevant parameters. Nonetheless, analogously to the analysis of a two-dimensional system in a magnetic field [9],  $k_z R$  may be regarded as the center of the electron's orbital motion which, for convenience here, is taken to be located in the  $z = 0$  plane. Note that the azimuthal magnetic field tends to project the electron's orbits in the plane containing the core-axis. Now, increasing  $k_z R$  means that the electron's wave function is compressed either against the core barrier or the parabolic walls of the electric potential of the host material. There is, however, a specific critical set of the relevant parameters, in particular,  $m$ ,  $B_s$  and  $k_z R$ , for which the corre-

sponding electron's wave function is least perturbed by the overall potential of the system. It is the electron's eigenvalue of this particular wave function which becomes the ground state. Fig. 1b shows the variation of the ground state ( $m = 0$ ,  $\ell = 1$ ) energy  $E_G$ , that is, the lowest energy eigenvalue, with  $B_s$ . The relevant parameters are:  $R = 50 \text{ \AA}$ ,  $\alpha = 0.025$  and the range  $0.00 \rightarrow 0.26$  of  $k_z R$  values. It is seen that this curve is made up of nearly straight line-segments, each corresponding to a differential  $\delta k_z R = 0.01$  of the electron's axial wave number. Reducing this differential will of course result in a much smoother curve. It is worth mentioning that the representation of the field by a constant value,  $B_s$ , somewhat overshadows the spatial inhomogeneity of the actual applied magnetic field. Nevertheless, for suitably chosen values of  $\alpha$  the electronic motion can be restricted to regions where the local magnetic field does not differ much from the value at the surface of the core.

### 3. Oscillator strengths

This section deals with the derivations of oscillator strengths for optical transitions, obtained within the dipole approximation. The radiation field considered is that of elliptically polarized light, taken in the form

$$\mathcal{E} = (\mathcal{E}_{0\rho} \cos \omega t, \pm \mathcal{E}_{0\phi} \sin \omega t, \pm \mathcal{E}_{0z} \sin(\omega t + \gamma)), \quad (14)$$

where  $\gamma$  is an arbitrary phase factor,  $\mathcal{E}_{0\rho}$ ,  $\mathcal{E}_{0\phi}$  and  $\mathcal{E}_{0z}$  are the amplitudes of the radiation field in the respective directions. In the above equation, the positive and negative signs correspond to right- and left-hand elliptical polarization, respectively. In cylindrical coordinates,  $\mathbf{r} = (\rho \cos \phi, \rho \sin \phi, z)$ , the interaction potential,  $W_{\pm} = e\mathcal{E} \cdot \mathbf{r}$ , in the dipole approximation takes the form

$$W_{\pm} = e[\rho \mathcal{E}_{0\rho} \cos \omega t \cos \phi \pm \rho \mathcal{E}_{0\phi} \sin \omega t \sin \phi \pm z \mathcal{E}_{0z} \sin(\omega t + \gamma)]. \quad (15)$$

Now, the term proportional to  $\mathcal{E}_{0z}$  of matrix elements,  $\langle m' \ell' | W | m \ell \rangle$ , vanishes because of the orthonormality of the wave functions characterized

by the same azimuthal but different radial quantum numbers. In particular, for circularly polarized light, that is,  $\mathcal{E}_{0\rho} = \mathcal{E}_{0\phi}$ , the matrix elements are found to satisfy the proportionality relationship

$$\langle n'\ell' | W | m\ell \rangle \propto I_{m\ell'm\ell} [\delta_{mm'+1} + \delta_{mm'-1}] \delta_{k_z, k'_z}, \quad (16)$$

where  $I_{m\ell'm\ell} = \langle \rho/R \rangle$  is the interaction integral given by

$$I_{m\ell'm\ell} = \int_1^\infty x^2 \gamma_{m\ell} \gamma_{m'\ell'} dx, \quad x \equiv \rho/R. \quad (17)$$

In the usual notation, the  $m\ell(m'\ell')$  quantum numbers refer to the initial (final) states of the electron. In the dipole approximation and specific to circular polarization, the selection rules for the allowed transitions are that  $m$  and  $m'$  must differ by unity and for the conserved axial wave number, that is,  $k_z = k'_z$ . As in similar evaluations in quantum well structures [19] or in quantum structures which possess cylindrical symmetry [14,18], the oscillator strength for optical transitions may be defined as

$$f_{m,n'} = 2 \frac{m_e}{\hbar} R^2 (E_{m'\ell'} - E_{m\ell})^2 I_{m\ell'm\ell}^2, \quad (18)$$

where  $m_e$  is the free mass of the electron. The oscillator strength may be regarded as an index of how strongly a system interacts with the radiation field. In view of potential device applications, the transitions of practical interest are those for which  $\Delta E = (E_{m'\ell'} - E_{m\ell})$  is in the far-infrared window of the electromagnetic spectrum, that is,  $\Delta E$  only a few milli-electron volts. Numerical results are illustrated only for the  $m = 0 \rightarrow m' = 1$  transitions, that is, involving the two lowest-order energy subbands, for both of which  $\ell = 1$ .

Fig. 2 shows the interaction integrals as functions of the magnetic field for the  $m = 0 \rightarrow |m'| = 1$ —optical transitions near a core of radius  $R = 50 \text{ \AA}$ . The additional parameters are  $\alpha = 0.025$  and the sequence  $\{-0.10, 0.00, 0.10\}$  of the  $k_z R$  values indicated there to identify the different curves. To be more specific,  $k_z R = 0.00$  for the thick solid curve, 0.10, for the smooth solid curve and  $-0.10$  for the dashed curve. It is seen

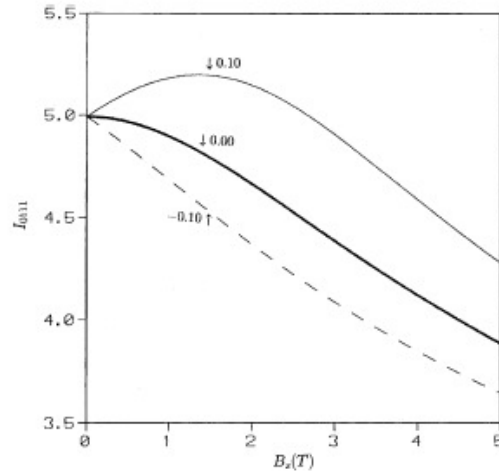


Fig. 2. The  $\ell = 1$  interaction integrals for the transitions:  $m = 0 \rightarrow m' = 1$ : as functions of the surface-value of the magnetic field near a core of radius  $R = 50 \text{ \AA}$  and such that  $\alpha = 0.025$ . The different curves are identified by the corresponding values of  $k_z R$ , which are: 0.00 for the thick solid curves,  $-0.10$  and 0.10, for the dashed and smooth solid curves, respectively.

that the curves for the interaction integrals corresponding to the three values of  $k_z R$  are triply degenerate at zero magnetic field. This degeneracy of the interaction integrals is lifted by the azimuthal magnetic field, with the curve corresponding to  $k_z < 0$  being depressed the most. While the curves for  $k_z \leq 0$  decrease monotonically with increasing  $B_z$ , the curve for  $k_z R = 0.10$  is lifted above these and in fact possesses a maximum. Arguably, for the relevant parameters of the system used, in particular the moderate values of  $k_z R$ , the electron's wave function corresponding to  $k_z R = 0.10$  is concentrated at relatively large radial distances. Note that the corresponding center of the electron's orbit is the furthest from the core axis of symmetry. By an earlier argument, the peak of the interaction integral occurs at a field when the electron's orbit fits well inside the overall potential profile. Apart from the initial rise of the curve corresponding to  $k_z R = 0.10$ , the general trend is that of the decreasing interaction integrals with the increase of the magnetic field. This is because as the field is increased, the electron's

wave function is increasingly tightened around the center of orbit and thereby reducing its spatial extent.

Fig. 3 shows the oscillator strengths for the  $|m'| = 1 \rightarrow m = 0$  optical transitions as functions of the surface-value of the magnetic field. The other relevant parameters:  $R = 50 \text{ \AA}$ ,  $\alpha = 0.025$  and the sequence  $\{-0.10; 0.00; 0.10\}$  of the  $k_z R$  values, are exactly as for Fig. 2. The values of  $k_z R$  are shown there to identify the different branches of the oscillator strengths. Just as for the interaction integrals, the azimuthal applied magnetic field lifts the triple degeneracy of the oscillator strengths. From a comparison of Figs. 2 and 3, it is seen that each of the corresponding branches of the interaction integrals and the oscillator strengths display more or less the same functional form in their variation with the field. This is because the energy separations between the relevant subbands, of course, corresponding to the same  $k_z$ , are more or less equal. For the relevant parameters used, stronger optical transitions are predicted for positive as opposed to negative values of the electron's axial wavenumbers. It is worth com-

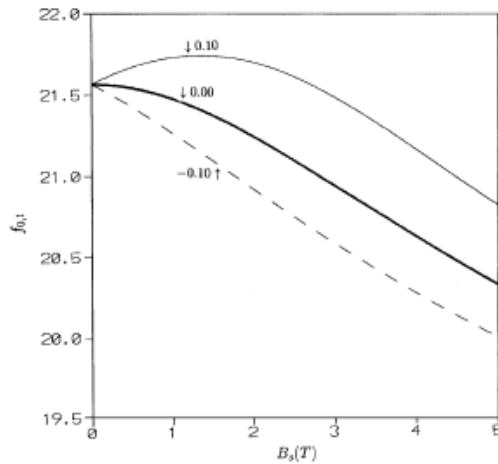


Fig. 3. The variations of the oscillator strengths with the surface-value of the magnetic field corresponding to exactly the interaction integrals shown in Fig. 2. The relevant parameters are as for the preceding figures;  $R = 50 \text{ \AA}$ ,  $\alpha = 0.025$ ,  $m = 0$ ,  $|m'| = 1$  and the values of  $k_z R = -0.10, 0.00$  and  $0.10$  shown there to identify the different curves.

menting that the explicit character of these optical transitions depends in a very critical way on the values of  $k_z > 0$  used. For example, there is a critical set of the relevant parameters of the system for which the argument of the radial function  $\zeta$  passes through zero and becomes complex. For exactly the set of parameters when  $\zeta = 0$ , the parameter  $a$  of the confluent hypergeometric function develops a singularity. The results obtained using the critical set of the relevant parameters of the system differ very drastically from those presented here. The results when both the parameter  $a$  and  $\zeta$  are complex cannot be adequately dealt with here and their discussion is not pursued further. Nevertheless, in the related experimental investigations, suitable values of  $k_z$  can effectively be selected through the application of an electric in the axial direction. The strength of the electric field required to give specific values of  $k_z$  can be deduced following the analysis of Das and Chakravarti [20]. It is also worth commenting that the results presented here are illustrative rather than predictive. This is on account of some of the unrealistic parameters used, in particular, the radius of the core. However, it is anticipated that more or less the same results shown here can be reproduced within a relatively small range of the field in the case of a larger radius. This should be the preferred balance of the relevant parameters, that is to say large  $R$  and small range of values of  $B_s$ , in the corresponding experimental investigations, for example, such as by Ashoori et al. [21]. Perhaps a more convenient representation of the applied field is through the dimensionless variable,  $f_{cs}$ . The electron energy subbands, for example, may be then be plotted as universal curves of  $E_{m'}/E_0$  versus  $f_{cs}$  where  $E_0 = \hbar^2/(2\mu R^2)$ . Note that in this representation, the radius scales both axes as  $R^2$  [17].

#### 4. Conclusions

The energy eigenvalues of a single electron confined near a current-carrying core as functions of the current-induced magnetic field have been obtained within the effective-mass approximation. The field was represented by,  $B_s$ , the value at the

surface of the core, which somewhat overshadows the spatial inhomogeneity of the actual applied magnetic field. This representation is therefore more appropriate for strengths of the electric potential such that electrons are confined in very narrow channels near the surface of the core. The azimuthal applied magnetic field was found to lift the double degeneracy of the  $k_z \neq 0$  energy subbands while that of the  $m \neq 0$  states is preserved. The situation is the other way around in the case of a parallel applied magnetic field. A further analysis involved the evaluations of oscillator strengths for optical transitions in the dipole approximation. In the dipole approximation and for circularly polarized light incident along the axis of the core, the selection rules for the allowed optical transitions were established to be: a) the final ( $m'$ ) and the initial ( $m$ ) azimuthal quantum number states must differ by unity; b) the electron's axial wave numbers of the final and the initial states must remain the same. The interaction integrals as well as the oscillator strengths for the transitions considered, in particular, for the  $k_z R = 0.00$  or  $\pm 0.10$  states, were found to be triply degenerate at zero magnetic field. The azimuthal magnetic field was again found to lift this triple degeneracy. For the relevant parameters used, the strongest optical transitions are predicted for positive axial wave numbers of the electron, moreover for weak magnetic fields. As noted previously, the radius of the core used here,  $R = 50 \text{ \AA}$ , is perhaps unrealistically small to support a measurable current. However, the results presented adequately illustrate the fundamental properties of the interactions investigated. In the corresponding experimental investigations, the current will have to be passed along a core of a sizeable radius. This has the added advantage that to reproduce the results

presented here, for example, a current of relatively low density would be required.

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