TEACHERS AND STUDENTS VIEWS PERTAINING TO THE TEACHING AND LEARNING OF SCHOOL GEOMETRY IN BOTSWANA JUNIOR SECONDARY SCHOOLS

Salome Maemo Mogotsi
University of Botswana
e-mail: Salome.mogotsi@mopipi.ub.bw
tel: 3552169

Kgomotso Getrude Garegae
University of Botswana
e-mail: GAREGAEK@mopipi.ub.bw
tel: 3552175

Sesutho Koketso Kesianye
University of Botswana
e-mail: Kesianyes@mopipi.ub.bw
tel: 3552164

Abstract

Geometry is one of the mathematical strands offered in school mathematics. Through the learning of geometry concepts students develop problem solving skills and become critical thinkers. Unfortunately performance on geometry questions by Botswana students is not good as shown by their performance in Trends in International Mathematics and Science Study 2003, 2007 and 2011. Good performance in geometry is very crucial because it is linked to other mathematical content and is a foundation of many science based careers. Mathematics teachers need to have the appropriate content and pedagogy in teaching geometry concepts. By so doing they will be able to explain the geometry concepts and select appropriate teaching methods. The use of dynamic geometry has been found to increase students' understanding of geometric concepts by making them 'less abstract'. Thus it should be incorporated in the teaching and learning of geometry concepts. This study sought to explore the challenges that teachers and students encounter in the teaching and learning of geometry concepts. Data was collected from form 3 students (n=198) and mathematics teachers (n=18). The data was collected using questionnaires (closed and open questions) and focus group interviews. The data from the closed questions was analyzed using SPSS and reported as frequencies. As for the interview questions they were analyzed by searching for common responses from the respondents which led to the development of themes. The results of the analyzed data indicate that indeed there are challenges faced by both teachers and students. They include shortage of resources and application of pedagogical content knowledge. The study recommends, in addition to others, that teacher training in the colleges of education should increase geometry content knowledge and incorporate use of dynamic geometry software.

Keywords: *Geometry; TIMSS; Dynamic geometry software*

Introduction

Geometry is one of the mathematical strands offered within the school curriculum, that is, from kindergarten all the way to secondary school and beyond. In fact, Piaget and Inhelder (1967), in their theory of Spatial and Geometrical Development, posited that spatial thinking begins at infancy and progresses as the child grows. Reasons as to why Geometry won its seat in the curriculum include its many connections with the real world; it develops problem solving skills, logical thinking, deductive and analytical thinking; connects different areas of mathematics (fractions, percentages, measures); its content is applied in careers such as construction, land surveying and architecture (Battista, Wheatley & Talsma, 1989; Arcavi, 2003). Moreover, when students engage in questions that involve geometry concepts it develops their spatial ability. Having good spatial ability is very important as it is positively related to achievement in mathematics (Aiken, 1971).

Clements (2003) indicates that although geometry is such an important mathematical concept early childhood and primary school teaching spends minimal time in teaching it. This is an indicator that as students progress to higher levels of learning they are most likely to underperform in it due to poor grounding in it. This scenario is worrisome for developing countries as they need their learners to uptake mathematically oriented careers. Such careers require freshman who are well grounded in mathematics. Good performance on geometry concepts is important because it contributes to the overall good performance in mathematics.

In the Botswana school mathematics curriculum, geometry is one of the mathematics content areas offered in school mathematics. That is, at the primary (standard 1-7) and secondary school levels (form 1- form 5) (Republic of Botswana, 1996 & 2010). This is consistent with international trends in which geometry is offered within the school curricula.

Performance in school mathematics has been reported to be declining at both primary and secondary school levels (Republic of Botswana 2007; Botswana Examination Council, 2014). These results are found in national and international school mathematics studies. Nationally two studies, Monitoring Learning Achievement Survey Project (2001) and Standard Four Assessment Report (2007) the pupils had a mean percentage score of 46.6% and 26.05% respectively on geometry questions. At the international level, Botswana students participated in Trends in International Mathematics and Science Study (TIMSS: 2003; 2007; 2011) in which its students showed poor performance in geometry questions. For instance, standard 6 students participated in TIMMS 2011 in which they had an average score 403.76 in Geometric Shapes and Measures. Botswana form one students participated in TIMSS 2003, 2007 and 2011 and they showed the least performance in geometry with average scores of 335 (3.9), 324.55(3.18) and 381(3.0) respectively. These TIMSS results are all below the TIMSS average scale average of 500 indicating that the students' performance is below per. Impoverished performance in geometry is a threat since it is the foundation in areas such as engineering, construction, astronomy, surveyors, mechanics and technology of which the country wants its student to take up careers in. This study therefore purports to explore challenges encountered by both teachers and students in the teaching and learning of geometry at Junior Secondary School level. It is hoped that its results will help in future teacher interventions.

Literature Review

The issue of performance in mathematics dates back many years. This is because of the significance that mathematics plays in our lives. Mathematics is at the heart of every activity that we engage in; be it formal or informal. As rightly indicated by Maliki, Ngban and Ibu (2009), mathematics is increasingly being used in science, technology and industry and hence its significant importance in the development of any developing country. This interrelationship of mathematics and development

... “is more related to the scientific and technological facets of man’s world more than any other aspect as it occurs and re-occurs in the physical and natural sciences...” (Maliki, Ngban & Ibu 2009: 131). As thus, the concern of performance in mathematics cannot be over emphasized. The issue of performance in mathematics will always be at the heart of all governments especially developing ones like Botswana. This is because they still need personnel in science related careers of which a solid mathematics background is a must especially in geometry. Areas such as mechanics, construction and surveying have many geometry topics.

International and national researchers have identified a number of different factors that have been found to contribute to under-performance in mathematics. These include the issues of teacher’s mathematical content knowledge (MCK), pedagogical content knowledge (PCK), teaching methods and resources, language, student beliefs, motivation and the learning environment (Major & Mangope, 2012; Adedoyin 2011; Mapolelo, 2009, Papanastasiou, 2008). The issues of teacher’s MCK, PCK, teaching methods and resources as well as language will be discussed further since they have a bearing on the other issues of student beliefs, motivation and learning environment in which mathematics is learned. Furthermore, these factors have been found to contribute immensely to the teaching and learning of school geometry.

Mathematics Content Knowledge

Mathematical content knowledge (MCK) of teachers is one of the most important factors that affect student achievement (Ball, Hill & Bass, 2005). There is a positive correlation between teachers’ content knowledge and their students’ mathematical success. This being the case it means that teachers need to be well grounded in their mathematics content. This is to ensure that when they are teaching they have a good understanding of the concept before they think of ways of teaching it. This therefore means it is a must for teachers to study geometry during their teacher training programme. Mathematics teachers of Botswana in the junior secondary schools are mainly diploma holders. During their teacher training programme they study geometry as a course for one semester (Republic of Botswana, 1996). This though may not be adequate as Grover and Connor (2000) indicate that generally prospective teachers study geometry when they are students at school and as only one course at college or university and as thus may not be very competent in teaching it. This is consistent with Hawk (1994) who reported that teachers had problems in teaching mathematics content that they had not encountered during their teacher training program. This means that content knowledge is paramount during teacher training. It must be adequate in terms of both depth and scope. Furthermore, Jones, Mooney and Harries (2002) report that pre-service and in-service mathematics teachers indicated that they have the least confidence in teaching geometry. This lack of confidence can have a negative effect on the students’ performance.

The Teacher Education and Development Study in Mathematics (Tatoo et al, 2012), indicated that Botswana future teachers’ mathematics content knowledge of being prepared to teach at junior secondary school was generally below par. This is an indicator that mathematics teachers at junior secondary school might be struggling with the content that they teach. This is detrimental as teachers’ mathematics content knowledge has been found to be one of the most important factors that affect students’ performance. Thus, teachers’ content knowledge in geometry needs to be highly grounded so that they have a thorough knowledge of geometry concepts far beyond the ones they will teach at the school level.

Pedagogical Content Knowledge

According to Adedoyin (2011) good teachers possess a substantial amount of pedagogical content knowledge (PCK) which is necessary for making students understand mathematical concepts. Such teachers have the ability to choose relevant activities to teach abstract concepts and are able to clarify concepts to students spot on. This is supported by Ball (2003) in Adedoyin (2011: 278) who indicates that “a teacher with good pedagogical content knowledge can unpack the mathematics into its discrete elements and can explain a concept or procedure at a level that includes the steps necessary for the students to make sense of the reasoning”. The expectation is that teachers at junior secondary school have adequate pedagogical content knowledge since they have been exposed to such during their teacher training programme. On the contrary though, Adedoyin (2011) indicates that there is still a need to improve on this knowledge as students perceived it as having an impact on their mathematical achievement. Tatoo et al (2012) also indicates that Botswana future teachers’ PCK was also low. This therefore means teachers need to have an in-depth of PCK. This would definitely work in favour of teaching of geometry since as mentioned earlier, varied relevant activities are necessary for the engagement of students in learning its abstract concepts.

Teaching Methods and Resources

Teaching methodologies have a direct effect on student achievement. Thus there is a need to use teaching strategies that foster student achievement. Such strategies include problem solving, investigational work, and application of mathematics to real world. These methods are consistent with those that have been recommended for teaching mathematics at the junior secondary school (Republic of Botswana, 2010). The recommended teaching methods include practical work, discussions, investigations and problem solving. Unfortunately the teaching of mathematics in Botswana secondary schools has been reported to be mainly teacher centred (Makagato & Mji, 2006; Mapolelo, 2009 and Chakalisa et al 2000). This scenario is still evident in our classrooms as reported by Major and Mangope (2012) who reported that student to student and/or student to teacher interaction is limited as students went through the teaching and learning process as passive recipients.

The teaching and learning of geometry can be compromised by using the teacher centred method since the development of concepts associated with it need to be done using student centred methods. Geometry topics have a good number of abstract concepts. Thus, the teacher centred method of teaching, which seems to be dominant in our mathematics classrooms will not help in the learning of these topics. Contrary to the norm, a geometry lesson should be dominated by students being engaged in individual or group activities using manipulatives (paper folding, geoboards, compasses, protractors e.t.c.) and/or computer applications (dynamic geometric software - DGS). Using manipulatives and DGS has been found to be helpful in assisting students learn geometry concepts (Scandrett, 2008 & Nkhwalume, 2013). Using DGS gives students an opportunity to engage in high level thinking. Using manipulatives and/or DGS is very paramount in the teaching and learning of geometric concepts. As thus, the success in teaching and learning of geometry has to be accompanied by using appropriate learner centred teaching methods and learning aids because of its abstract nature. Unfortunately in Botswana schools mathematics teachers have little to no access to computers making teaching mathematics using mathematical software becomes futile (Nkhwalume, 2013).

Language

Different researchers (Korhonen, Linnanmäki and Aunio, 2012; Garegae 2007; Jamisson, 2000 and Pimm, 1987) have indicated that there is a link between students’ mathematical achievement and proficiency in the language of instruction. Low mathematical achievement has been found to be directly linked with incompetency in the language of instruction. Students who are taught mathematics in a second language have to struggle with both the medium of instruction and specialized language: definitions, syntax, symbols (Jamisson 2000 & Pimm 1987). This can negatively affect students’ understanding of such terms found in both languages. Bell (2003) in Garegae (2007:1) indicates that students whose instructional language (English) is their mother tongue have been found to struggle with the "... highly specialised mathematics terms with varied meaning in spoken everyday English…". As can be imagined the matter becomes worse with those whose instructional language is a second or third language. Students' achievement is compromised by the language barrier. As a result there is a tendency for teachers to code switch between English and Setswana during lesson instruction in order to clarify concepts and redirect learning (Garegae, 2007). As thus classroom discourse is conducted in two languages. This type of set up does not develop students' communicative skills and understanding mathematical concepts, more so geometrical concepts.

Research Questions

For the purpose of this research paper the following research questions are raised:

1. What are the views of teachers and students in teaching and learning of geometry?
2. What challenges do teachers meet when teaching geometry?
3. What challenges do students meet when learning geometry?

Research Methodology

The research design for this study adopted a triangulation approach because the respondents answered a questionnaire and were engaged in an interview. The purpose of triangulation was to corroborate the collected data. This is in line with Creswell (2014) who indicated that the use of various multiple instruments helps to reduce bias and weakness, thus generating more accurate data. Since the data instruments collected both quantitative and qualitative data, the research design falls within the qualitative and quantitative research approaches. This study employed a descriptive survey method which was intended to provide a numeric description of the respondents' sample. Since the data was collected at one point it makes the study to also be a cross sectional survey (Creswell 2014).

Sample

The sample for this study included 18 mathematics teachers and 198 form three students from six junior secondary schools in the southern region of Botswana. Form three students were used because they had covered the syllabus substantially. There is a total of 10 educational regions in Botswana. The region, students and mathematics teachers were selected with equal probability of selection using a single stage systematic random sampling. This method of sampling allowed each member of the population to have an equal chance of being part of the sample (Ary, Jacobs & Sorensen, 2010; Gay& Airasian 2011).

Instrument and Procedures of Validation and Reliability

Data for the study was collected by the researchers using four instruments; teacher questionnaire and interview schedule, student questionnaire and interview schedule. The questionnaires both had closed and open ended questions. The closed ended questions required the respondents to rate the geometry objectives as easy or difficult to teach or learn. As for the interview guide it was designed to get data mainly pertaining to classroom discourse.

The validity of the instruments was checked by experienced mathematics educators in the University of Botswana. A trial test of the questionnaires was done using in-service students in Bachelor of Education (Secondary) at the University of Botswana. Furthermore, a pilot test of the questionnaires was carried out by using students and teachers from a school in Gaborone. Some modifications on the instruments was done using feedback from the trial and pilot tests. The reliability of the Likert scale questionnaire items was examined using Cronbach's alpha. All items were found to be reliable as they all scored alpha coefficient of 0.96 and above. This indicates that the internal consistency of the items was reliable since their alpha coefficient was greater than 0.7 (Bruin, 2011). Checking validity and reliability of the instruments ensures that the instruments measure as accurately as possible what they are expected to measure (Gay, Mills & Airasian, 2009).

Data Collection

The data collection process took approximately two weeks. A total of 25 teacher questionnaires and 200 student questionnaires were distributed. The response rate was 72% and 99% for teacher questionnaire and student questionnaire respectively. The focus group interviews were two: one for the teachers and the other for students. The focus group for the teachers comprised of five mathematics teachers whereas that of students consisted of eight students. The interview sessions took approximately 40 minutes. The participants had filled in consent (teachers and parents on behalf of the students) and assent forms (students). Participant willingness should be sought prior to collecting data (Ary, Jacobs & Sorensen, 2010; Gay& Airasian 2011).

Data Analysis

The data analysis was done using statistical package for social sciences (SPSS) for the quantitative data of the closed ended questions. SPSS was used for data entry, data cleaning and running descriptive statistics. The descriptive statistics was used to generate tables of frequencies. As for the qualitative data it was analysed by searching for common responses from the respondents which led to the development of themes. The data with similar themes was put together through open and axial coding (Strauss, 1987). The key findings were then reported in narrative form.

Results and Discussions

The findings of the results are discussed according to the research questions as follows:
Research Question 1:
What are the views of teachers and students in teaching and learning of geometry?

Both students and teachers understand the importance of learning geometry concepts as indicated in Table 1. They also indicated that there were some geometry concepts that they deemed to be either be difficult to teach or learn as indicated in Tables 2, 3 and 4.

Table 1: Students' and teachers' views on the learning of geometry concepts

Students' views	Teachers' views
Will use it senior secondary school	It is important for use in the future in science based subjects like engineering
Apply knowledge learnt in geometry in other subjects. For example in Design and Technology we do constructions and angles which are covered in geometry	Foundation for other geometry topics to be taught at senior secondary school
	Can be used in other subjects like design and technology and agriculture

Table 2: Frequency on students and teachers view of form 1 geometry topics as easy

Topics and Sub- topics	Content	Students Frequency Easy (%)	Teachers frequency Easy (%)
Geometrical constructions	draw line segments, angles, parallel and perpendicular lines	63.6	72.2
	construct line segments, angles, parallel lines, perpendicular lines	46.5	55.6
	construct angle and perpendicular bisectors	43.4	77.8
Angle Properties	calculate unknown angles involving adjacent angles on a straight line, vertically opposite angles, and angles at a central point formed with a common vertex	43.9	88.9
	calculate unknown angles involving corresponding angles, alternate angles, interior angles, complementary angles, supplementary angl (co-interior angles) formed by parallel lines and cutting lines	42.9	94.4
Polygons	describe both line and rotational symmetries of polygons; calculate sum of t interior angles of polygons	55.6	66.7
	investigate the sum of the exterior angles of polygons; calculate the number of sides of polygons	58.6	77.8
	calculate the missing angles of polygons by applying angle properties of polygons	54.0	83.3
Coordinate geometry	Plot points given coordinates in any of the four quadrants.	80.8	83.3
	plot coordinates in any of the four quadrants and join them to form different shapes	63.2	83.3
	plot coordinates in any of the four quadrants and join them to draw lines of the form x = a and y = a	49.5	83.3
Transformation geometry	reflect plane figures along x=a and y=a	42.9	72.2
	draw and interpret column vectors	42.9	66.7
	add and subtract column vectors	58.6	83.3
	multiply a column vector by a scalar	39.7	83.3
	translate plane figures on a coordinate grid using different translation vectors	39.4	72.2
	draw an enlargement/reduction of a plane figure on a coordinate gri using positive scale factors of both whole numbers and fractions	38.4	61.1
	rotate plane shapes on a coordinate grid using any centre of rotation and angle of rotation(s) which are multiples of 90^o	47.0	50.0
	identify and describe reflection, translation, enlargement and rotation fully	52.5	50.0

Table 3: Frequency on students and teachers view of form 2 geometry topics as easy

Topics and Sub- topics	Content	Students Frequency Easy (%)	Teachers frequency Easy (%)
Geometrical constructions	construct triangles from the given data	52.5	66.7
	identify congruent triangles and their corresponding measures	40.9	61.1
	identify similar triangles and their properties	51.5	66.7
	construct quadrilaterals from the given data	41.9	61.1
Angle Properties	calculate unknown angles of triangles using properties of triangles which include sum of interior angles in a triangle, base angles of isosceles triangle, angles in an equilateral triangle, and sum of exterior angles in a triangle	53.0	94.4
	calculate unknown angles of quadrilaterals using angle properties of square, rectangle, parallelogram, rhombus, kite and trapezium	51.0	72.2
Polygons	construct regular polygons from the given data	44.9	55.6
	use reflection, translation, and rotation to draw congruent polygons on a coordinate grid	42.4	38.9
	use enlargement and reduction to draw similar polygons on a coordinate grid	40.4	33.3
Coordinate geometry	draw graphs of the form $y = mx + c$ to represent linear relationships	57.5	83.3
	use a spreadsheet to draw graphs of the form: $y = mx + c$	41.4	55.6
	find the equation of a straight line of the form: y = mx + c	45.9	72.2
	draw graphs of the form $y = ax\ 2 + bx + c$ to represent to represent quadratic relationships	26.3	72.2
Transformation geometry	reflect shapes on a coordinate grid using line of reflection of the form $y = mx + c$	28.3	66.7
	draw an enlargement and reduction of shapes in a coordinate grid using positive or negative scale factors for whole numbers and fractions	28.7	44.4
	rotate plane shapes on a coordinate grid using any centre of rotation and angles of rotation being multiples of 10º	31.9	50.0
	identify and describe reflection, translation, enlargement and rotation fully	42.4	55.6

Table 4: Frequency on students and teachers view of form 3 geometry topics as easy

Topics and Sub- topics	Content	Students Frequency Easy (%)	Teachers frequency Easy (%)
Coordinate geometry	calculate the distance between two points	51.6	83.3
	calculate the coordinates of the midpoint of a line segment given coordinates of its endpoints	33.9	77.8
	calculate the coordinates of the endpoint of a line segment given coordinates of the midpoint and one end point	19.7	77.8
	solve problems involving the applications of distance between two poir and midpoint of the two end points of a line segment	24.7	55.6
Transformation geometry	draw combined transformations involving up to three different types of transformations	19.2	44.4
	identify and describe fully combined transformations involving up to three different types transformations	18.2	50.0
	solve problems involving combined transformations	22.3	38.9
Triangles	derive the mid-point theorem by investigation	14.1	61.4
	derive the converse of the mid-point theorem by investigation	14.1	44.4
	solve problems involving the mid-point theorem	20.7	55.6
	solve problems involving the converse of the mid-point theorem	15.6	44.4
	calculate unknown sides and angles using properties of congruent triangles	33.4	66.7
	calculate unknown sides using properties of similar triangles	33.3	77.8
	solve problems involving properties of congruent triangles and similar triangles	27.8	72.2
Pythagorean Theorem And trigonometric ratios	derive Pythagorean theorem by investigation	29.3	61.1
	calculate the unknown side of a right-angled triangle using the Pythagorean theorem	35.3	88.9
	use Pythagorean theorem to solve problems related to real life situation	30.3	83.3
	identify opposite, adjacent, hypotenuse sides of a right-angled triangle, with reference to the given angle	37.9	94.4
	calculate the three Trigonometric ratios of sine, cosine and tangent in a right-angled triangle	63.6	66.7
	use the three Trigonometric ratios and a calculator to calculate the unknown angle and side in a right angled triangle	31.8	55.6
Plans,	draw front, side and plan elevations of the same shape and house	28.3	50.0

Elevations and bearings	find a point's position using direction/bearings, distances and/or angles in a given journey	24.2	55.6
	use Pythagorean theorem and trigonometric ratios to solve problems involving angle of elevation, angle of depression and bearings	11.6	33.3

The results indicate that objectives that were deemed difficult to teach by the teachers were mainly from the topic Transformation Geometry. For example rotating shapes on a coordinate grid, identifying and describing reflection, translation, enlargement and rotation fully (form one), Use enlargement and reduction to draw similar polygons on a coordinate grid (form 2) and Solve problems involving combined transformations (form 3) in which 50%, 66.7% and 61.1%) of the teachers indicated respectively that they were difficult to teach. If teachers are having difficulty then what about the students? The results are not surprising because students also indicated that objectives under Transformation Geometry are difficult to learn. This is consistent with Adedoyin (2011) who indicated if teachers are lacking in pedagogical content knowledge their students are unlikely to succeed.

The objectives that the teachers indicated as easy to teach and students indicated as easy to learn were mainly low order objectives. For example, 94.4 % of the teachers indicated that teaching calculating unknown angles (form one), calculate unknown angles of triangles using properties of triangles which include sum of interior angles in a triangle, base angles of isosceles triangle, angles in an equilateral triangle (form 2), and sum of exterior angles in a triangle and Identify opposite, adjacent, hypotenuse sides of a right-angled triangle, with reference to the given angle (form 3) they were easy teach. As for the students, 80.8% and 57.5 % respectively indicated that Plot points in any of the four quadrants (form one) and Draw graphs of the form $y = mx + \mathrm{c}$ to represent linear relationships. Being able to perform best mainly on low order questions does not augur well with the aspirations of the country in which they want their students to enrol mathematics related careers. For this to happen students must be able to perform well on high order objectives.

Research Question 2:
What challenges do students meet when learning geometry?

The students indicated the following (Table 5) as challenges they encountered by students when learning geometry. The responses were categorized under the categories student related, teacher related and language.

Table 5: Students challenges in learning geometry concepts

MAJOR CATEGORIES	**CHALLENGES**
	ATTITUDE Wrong attitude towards the topic. Told oneself that the concept to be taught is difficult before it had been taught; Do not pay attention in class; Do not like the topics, in fact do not like mathematics. It is boring; Lack of studying or regular revision. Do not give oneself time to revise.
	MATERIALS

	Lack of materials in the form of mathematics set and textbooks.
STUDENT RELATED	**CONTENT** New content which is hard to remember especially at form 2 and form 3; Some topics have not been taught; Drawing and construction difficult. ; Too many sub topics (many concepts) which are not easy to learn. They are confusing and tricky even if research on them. Many ways of solving which is confusing; Long topics but little time spent on them. ; Transferred to another school and did not do some of the topics; Many difficult and confusing formulae and equations to be grasped. Too many applications and calculations. The lesson ends without understanding since explanations are too difficult.; Time consuming; More concentration needed
	UNDERSTANDING Take long time to understand because the concepts are difficult. They are for university level. Difficult to understand teacher but classmate easy. Not related to life making it difficult to understand; Slow learner. Need more time for understanding since I forget easily; Teacher ask if understand just say yes but not true; Lack of confidence, too shy to ask in class or ask for help; Absent- miss lesson it is difficult to understand
TEACHER RELATED	Not free in class, scared to ask teacher where we don't understand. Scared to ask teacher because she/he is angry or in bad mood or stressed; Teacher explains concepts very well; Teacher misses lessons or come very late then rushes through the lesson.
LANGUAGE	Mathematics language is difficult, fancy, and different from English language. If simple English was used I would understand the topics. Words like hypotenuse, bisect, factorise not commonly used in everyday life; Mathematical English and scientific English are difficult especially for slow learners. English words in mathematics difficult to understand since use huge difficult terminologies which we forget easily; Some words meet only in the test and becomes problematic when answering; Do not understand English; Make mathematics an option.

The challenges faced by students are diverse. Although this the case they are mainly pointing to the pedagogical content knowledge (pck) of the teacher. This is because the students indicate challenges of understanding the content although they acknowledge that they have a negative attitude towards mathematics. The teacher must be able to address these problems through the PCK since it equips them with knowledge and skills that enable them to make their students understand mathematical concepts, instil in them a positive attitude towards mathematics and make students understand the mathematical terminologies employed (Garegae 2007, Pimm 1987 & Adedoyin,2011) to name a few.

Research Question 3:
What challenges do teachers meet when learning geometry?

The responses from teachers are summarised in Table 6 below. They are reported under the headings Resources, Teacher-student ratio, Students progression, ICT, Teacher training, Language, Primary School and Marking national examinations.

Table 6: Teachers' challenges in teaching geometry concepts

MAJOR CATEGORIES	CHALLENGES
RESOURCES	Lack of materials in the form of mathematics set, textbooks, quad boards and geoboards. Since students do not have mathematical sets they do not do or practice questions dealing with constructions e.g. Transformations; Government policy is that each student must be given 1 textbook, 1 notebook and mathematics set but they are only given notebooks. As for textbooks they are supposed to use those that were used by students in the upper class. Students share textbooks which poses many problems. No geoboards.
TEACHER-STUDENT RATIO	The teacher student ratio is 1: 40. This is too large especially for geometry concepts which must be taught in a practical manner or using hands on activities
ICT	No mathematics lab; accessing computer lab is difficult thus do not use mathematics computer programmes (GSP/GeoGebra) to teach geometry concepts; write GeoGebra on cds and give to students to practice on their personal computers. No time as syllabus is congested, no computers; Some teachers attended workshops on using ICT to teach mathematics concepts especially geometry ones but training gone to waste
TEACHER TRAINING PROGAMME	Professional studies (pedagogy) equips us with enough knowledge to teach mathematics but unfortunately cannot apply the knowledge optimally in the schools because of limited resources; Geometry content offered at colleges of education is offered for approximately half a semester. This is not adequate; Did not undertake any ICT at college please include.
LANGUAGE	A lot of code switching (English – Setswana= local language); mathematics questions have a lot of wording. Students then fail to understand them. Application questions students fail them not only in geometry but even other mathematical strands. English language is problematics

The challenges indicated by the teachers are mainly inclined to PCK. For example, teachers indicated that they struggle to teach students due to their students' incompetence in mathematics. But teacher training is supposed to equip teachers with knowledge and skills of teaching mathematics and handling students. There is a possibility that PCK covered in the teacher training programme may not be adequate as indicated Tatoo et al, (2012) and Adedoyin (2011). The teachers indicated that they did not cover enough geometry content when studying at the colleges. This can be detrimental because there is need for teachers to have a good foundation in geometry (Ball, Hill & Bass, 2005; Hawk, 1994) for them to confidently teach geometry concepts. Furthermore, teachers indicated that they do not teach geometric concepts using dynamic geometry software (DGS) because of lack of access to computers. This is consistent with Nkhwalume (2013) who indicated that mathematics teachers have limited or no access to computers.

Conclusion

Teachers and students both recognise the importance of teaching and learning of geometry concepts. This is positive because when one addresses issues related to difficulties in teaching and learning of geometry concepts they have somewhere to start. There are some challenges experienced by both students and teachers in the teaching and learning of geometry concepts. These challenges need to be addressed in order to improve students' performance on questions related to geometry concepts. This is very important because geometry is a mathematical strand that equips learners with problem solving skills and makes them to become critical thinkers, an asset that every learner must have. Moreover, geometry is applied in other school subjects and it is a foundation for some tertiary education subjects.

Recommendations

The findings of the study indicate although the teachers generally indicate that they are well versed with the teaching of geometry concepts they also have some shortcomings. The shortcomings are indicated by both students and teachers. Therefore, the study recommends:

- Teacher training must expose student teachers to more content on geometry topics especially at teacher training colleges. In addition a course on integration of ICT be introduced in the colleges of education curriculum.
- In-service training must be mounted to assist teachers on how teach mathematical concepts they deemed difficult to teach. The training must also include the use of dynamic geometry software in the teaching of geometry concepts.
- Each school must be given GeoGebra because it is a freeware. Where schools can afford to purchase relevant software to teach geometry concepts they should be encouraged to do so.
- Each student must be provided with a mathematical set and textbooks as per government expectations. In order to ensure that the resources (especially mathematical sets) are readily available they can be given to mathematics teachers for safekeeping.

References

Adedoyin, O.O 2011. The impact of teachers" in-depth pedagogical mathematical content knowledge on academic performance: As perceived by Botswana junior secondary school pupils. *European Journal of Educational Studies*. 3, 2:

Aiken, L. R. 1971. Intellective variables and mathematics achievement: Directions for research. *Journal of School Psychology* 9: 201-212

Arcavi, A. 2003. The role of visual representation in the learning of mathematics. *Educational Studies in Mathematics* 52, 3:215-241.

Ary D., Jacobs L. C., Sorensen C. 2010. *Introduction to Research in Education,* 8th edition. Belmont, CA: Wadsworth

Ball, D.L., Hill, H.C. & Bass, H. 2005. Knowing mathematics for teaching: Who Knows mathematics well enough to teach third grade, and how can we decide? *American Educator,* 29, 1:14-46.

Battista M.T., Wheatley, G.W. & Talsma, G. 1989. Spatial visualization, formal reasoning, and geometric problem-solving strategies of preservice elementary teachers. *Focus on Learning Problems in Mathematics,* 11, 4:17-30.

Botswana Examination Council. 2007. *Trends in International Mathematics and Science Study.* Gaborone: Government Printers and Publishing Services.
Botswana Examination Council 2014. *Summary of results*. Retrieved 22nd August 2017 at http://www.bec.bw
Chakalisa, P. A., Kyelve, I. J. & Matongo, K.M. 2000. Assessment in Botswana school mathematics: Issues and perspectives. *Mosenodi.* 8, 2: 47 – 58.
Coad, L. 2006. Paper Folding in the Middle School Classroom and Beyond. *Australian Mathematics Teacher,* 62,1:6-13
Creswell J.W. 2014. *Research Design*. London: SAGE Publications Ltd.

Hanson, W. E., Plano Clark, V. L., Petska, K. S., Creswell, J. W., & Creswell, J. D. 2005. Mixed methods research designs in counselling psychology. *Journal of Counselling Psychology, 52*, 2:224-235

Garegae, K. G. 2007. On code switching and the English language proficiency: The case of mathematics learning. *International Journal of Leaning* 14, 3: 233-237.
Gay, L.R., Mills, G.E. & Airasian, P.W. 2011. *Educational Research,*10th edition. New York: Pearson Education International

Grover, B.W., & Connor, J. 2000. Characteristics of the college geometry course for preservice secondary teachers. *Journal of Mathematics Teacher Education*. 3, 1:47–67

Hawk, P.P. 1994. *Making a difference: Reflections and thoughts of first year teachers.* Greenville, NC: East California University.
Jamisson, R. E. 2000. Learning the language of mathematics. *Language and Learning across Disciplines.* 4, 1: 45-54.
Jones, K., Mooney, C. and Harries, T.: 2002, Trainee primary teachers' knowledge of geometry for teaching. *Proceedings of the British Society for Research into Learning Mathematics*, 22, 1&2: 95-100.
Korhonen, J. Linnanmäki, K. & Aunio, P. 2012. Language and Mathematical Performance: a Comparison of Lower Secondary School Students with Different Level of Mathematical Skills. *Scandinavian Journal of Educational Research,* 56, 3: 333–344.
Major, T. And Mangope, B. 2012. The constructive theory in mathematics. The case of Botswana primary schools. *International Review of Social Sciences and Humanities*. 3, 2:139-147.
Makgato, M. & Mji, A. 2006. Factors associated with high school learners‟ performance: a spotlight on mathematics and physical science. *South African Journal of Education.* 26, 2:253-266.
Maliki, A., Ngban, A. & Ibu, J. 2009. Analysis of students' performance in junior secondary school mathematics examination in Baleysa state of Nigeria. *Stud Hom Sci,* 3, 2:131-134
Mapolelo, D.C. 2009. Students‟ experiences with mathematics teaching and learning: listening to unheard voices. *International Journal of Mathematical Education in Science and Technology,* 40, 3:309-322.
Nkhwalume, A.A. 2013. The challenges of integrating ICTs into the mathematics curricula in the SADC region: the case of Botswana. *Academic Research Journal.* 4, 2:332-337
Papanastasiou, C. (2008). A residual analysis of effective schools in teaching mathematics. *Studies in Educational Evaluation.* 34, 1:24-30.
Piaget, J. and Inhelder, B. 1967. The Child's Conception of Space. New York: Norton.
Pimm, D. 1987. *Learning Mathematics: Issues theory and classroom practice*. London: Cassel Education.
Republic of Botswana. 2010. *Junior secondary school mathematics syllabus*. Gaborone:

Government Printers.
Republic of Botswana. 1996. *Senior secondary school mathematics syllabus*. Gaborone: Government Printers.
Republic of Botswana. 2010. *Primary school mathematics syllabus*. Gaborone: Government Printers.
Scandrett, H. 2008. Using Geoboards in Primary Mathematics: Going… going... gone? *Australian Primary Mathematics Classrooms*. 13, 2:29-32.
Strauss, A. 1987. *Qualitative Analysis for Social Scientist*. Cambridge: Cambridge University Press
Tatto, M., Shewille, J., Senk, S., Ingvarson, L., Rowley, G., Peck, R., Banlov, K., Rodriguez, M., and Reckase, M. (2012). *Police, Practice and Readiness to Teach Primary and Secondary Mathematics in 17 Countries.* Amsterdam: Multicopy.